THE

Preſent State

OF THE

REPUBLICK

OF

LETTERS.

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For AUGUST, 1736.

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THE

THE

PRESENT STATE

OF THE

Republick of Letters.

For AUGUST 1736.

ARTICLE V.

REMARKS *on the* Conſiderations *relating to* Fluxions, &c. *that were publiſhed by* Philalethes Cantabrigienſis *in the* Republick of Letters *for the laſt month.*

IN theſe *Remarks*, for brevity, the words of *Philalethes* are not tranſcribed, but the *Sections* and *Paragraphs*, wherein they are contained, are particularly quoted.

The firſt four Sections contribute nothing towards determining the points in queſtion.

Sect. V. §. 1—4. By the idea of fluxions Mr. *Robins* does not mean the doctrine of fluxions, but only one part of that doctrine; for it is expreſly ſaid in the *Republick of Letters* for *October*

There is an apparent discrepancy at this point.

The pages are either missing or the pagination is incorrect.

The filming is recorded as the book is found in the collections.

laſt, pag. 253. that the doctrine of fluxions conſiſts of two parts, the form of conception there deſcribed (that is the idea of fluxions) and the method of applying it to the ſolution of mathematical problems.

Again, Mr. *Robins* does not conclude, that the method of fluxions is abſolutely diſtinct from that of firſt and laſt ratios, only becauſe Sir *Iſaac Newton* had formed his idea of fluxions before he had invented the other method, but alſo becauſe that other method is no otherwiſe made uſe of in this than for demonſtrating the proportions between different fluxions.

§. 5—8. Here it is attempted to be proved, that one of theſe methods could not poſſibly be invented before the other; becauſe Sir *Iſaac Newton* has in his writings made uſe of that of prime and ultimate ratios in demonſtrating propoſitions in the other doctrine; whereas to make this a concluſive argument, it was neceſſary to ſhew, it were impoſſible to form theſe demonſtrations by any other means; but it appears in fact Sir *Iſaac Newton* did at firſt content himſelf with ſuch demonſtrations, as the method of indiviſibles did afford; and Mr. *Robins* has ſhewn, that theſe propoſitions may be perfectly demonſtrated another way by exhauſtions.

§. 9. Becauſe *Philalethes* ſees not the uſe of this diſtinction, is that a proof it is of no importance? How could *Philalethes* imagine the letter S to be inſerted through inadvertency, when that is to ſuppoſe the title page printed before the book was writ; for theſe two methods are as much diſtinguiſhed in the book itſelf, as in any thing Mr. *Robins* has ſince publiſhed.

§. 11, 12. Has not Sir *Iſaac Newton* ſufficiently diſtinguiſhed them in delivering his method of prime

prime and ultimate ratios by itſelf at the beginning of his *Principia?* Has he not ſufficiently diſtinguiſhed them, when at the end of his *Introduction* to the *Quadratures*, he ſpeaks of determining the fluxions of quantities by the method of prime and ultimate ratios?

§. 13. Sir *Iſaac Newton* does not intermix his ſimple and plain deſcription of fluxions with the terms uſed in the doctrine of prime and ultimate ratios; for his deſcription of fluxions is contained in the two firſt paragraphs of his *Introduction* to the *Quadratures*, in which no terms of the other doctrine occur: the following part of the *Introduction* is not to inſtruct us in the idea, he deſigns to be annexed to the word fluxion, but only contains directions how by the method of prime and ultimate ratios to find the proportions between different fluxions. Therefore it ſtill appears, that the idea of fluxions has no relation to the doctrine of prime and ultimate ratios, and that the doctrines are abſolutely diſtinct from each other. For is not the fifth propoſition of the firſt book of *Euclide* abſolutely diſtinct from the fourth, though the fourth is the means whereby the fifth is demonſtrated. By the argument here made uſe of the doctrine of fluxions might as well be proved not to be diſtinct from the ancient method of exhauſtions; ſince Mr. *Robins* has ſhewn how to apply that method to the determining the proportions of fluxions.

§. 14. Here the diſtinction made at the beginning of Mr. *Robins*'s laſt Diſcourſe is not attended to; where it is obſerved, that the ſame method of calculation is applicable to both methods. And if theſe two methods being equally ſubſervient to the ſame method of computation makes them the ſame, then they are alſo the ſame with the differential method of *Leibnitz*.

Sect. VI. §. 1—4. In reality the words here quoted from Sir *Iſaac Newton* are the moſt unguarded, that great man has uſed upon theſe ſubjects, and upon theſe *Philalethes* has unhappily built his whole explanation of prime and ultimate ratios. If theſe expreſſions are unintelligible or incorrect, what juſtification will it be to *Philalethes* not to have diſcovered their imperfection?

§. 5, 6. Why will *Philalethes* ſo poſitively aſſert, that Sir *Iſaac Newton*, when he wrote his *Analyſis*, did not there proceed upon the principles of indiviſibles, ſince they are apparently uſed in that treatiſe, and it is affirmed in the account of the *Commercium Epiſtolicum*, that he was accuſtomed to make uſe of indiviſibles at that time, and the terms relating to them, in the ſame ſenſe with *Cavalerius* their original author. But it ſeems this account was writ after Sir *Iſaac Newton* had corrected that faulty doctrine; but is that a reaſon, why in giving an account of what he had done before, he ſhould either diſguiſe or conceal the truth. It is alſo, it ſeems, in this account declared, that Sir *Iſaac Newton* continued to uſe the letter *o* for an infinitely ſmall quantity, even to the time, when that account was writ; but Mr. *Robins* has never denied, that Sir *Iſaac Newton* might ſometimes have allowed himſelf in the uſe of indiviſibles in the inveſtigation of problems, even to his dying day. Mr. *Robins* has ſhewn, where he uſed them in his *Principia*. Here I muſt take leave to inform *Philalethes*, that he does not underſtand, what uſe Sir *Iſaac Newton* made of the letter *o*; for he aſſerts, that it denotes the only infinitely ſmall quantity uſed in his calculus; but a mathematician ought to have known, that of one infinitely ſmall quantity only no uſe at all can be made; it is only here meant, that

that Sir *Iſ. Newton* uſed no other ſymbol to expreſs an infinitely ſmall quantity, nay this means only no other ſimple ſymbol; for in the paſſage juſt quoted by *Philalethes* at §. 5. it is ſaid *prick'd letters never ſignify moments, unleſs when they are multiplied by the moment* o *either expreſſed or underſtood to make them infinitely little, and then the rectangles are put for moments.* Now according to *Philalethes* all moments are infinitely little quantities; therefore when Sir *Iſaac Newton* uſed any infinitely little quantities at all, he really uſed as many different infinitely little quantities as he conſidered moments, though he did not mark them by different ſimple characters, as others had been accuſtomed to do. Beſides, Mr. *Robins* has ſhewn, that Sir *Iſaac Newton* did not always intend an infinitely little quantity by the letter *o*. See *Republick* for *April* laſt, pag. 324. and this Diſcourſe of *Philalethes*, §. 10. of the preſent ſection.

§. 7, 8. Surely *Philalethes* has more reſpect for Sir *Iſaac Newton*'s memory than to inſinuate, that, when he was defending his method, he was not careful to give a true and unexceptionable deſcription of it.

§. 9--12. Here in relation to the words *volui oſtendere, &c.* I will only ask *Philalethes*, whether Sir *Iſaac Newton* does not mean, that he intended in the *Introduction* to the *Quadratures*, which is cloſed by theſe words, to ſhew, that it was not neceſſary infinitely ſmall quantities ſhould be introduced into geometry; and whether he has executed that deſign he propoſed? If he has, the doctrine of prime and ultimate ratios made uſe of throughout that Introduction does not imply the conſideration of any infinitely ſmall quantities.

§. 13, 14. Becauſe Sir *Iſaac Newton* thought fit to own, that it is poſſible to avoid error in the uſe of the principles of indiviſibles with proper

circumſpection; does it therefore follow, that he himſelf made uſe of infinitely ſmall quantities at the very time he undertook to ſhew there was no neceſſity for ſo doing?

Sect. VII. §. 1—4. Any given difference does not mean merely any aſſignable difference, but any aſſignable difference that ſhall be given. Sir *Iſaac Newton* at the end of his *Analyſis* uſes the like phraſe exactly in the ſenſe Mr. *Robins* contends for; for he there obſerves, that by adding continually freſh terms to the ſeries there diſcourſed of, *ultimus terminus* (*per* 1. 10. *Elem.*) *tandem evadet minor quavis data quantitate*; *& prorſus evaneſcet, ſi opus infinite continuatur*. Now it is impoſſible to continue the ſeries till you find a term leſs than any aſſignable. And Sir *Iſaac Newton* himſelf expreſsly diſtinguiſhes the diminution he firſt ſpeaks of from the vaniſhing of the term, (whereby alone it can become leſs than any aſſignable,) ſince he adds, that for this purpoſe the ſeries muſt be infinitely continued.

§. 6, 7. In the long ſtring of phraſes here collected, they are ſo far from being all deſigned to expreſs the ſenſe of *data quavis differentia*, that they have not all the ſame meaning one with another; *aſſignable difference*, or *difference that can be aſſigned*, means *differentia quæ dari poteſt*; in a tranſlation of the fifth phraſe not one of theſe words ought to be uſed.

§. 9. This is a wrong repreſentation of Mr. *Robins*'s meaning, he does not expound the words in queſtion by *any difference how minute ſoever, that can be aſſigned*, but has repreſented the ſenſe of the *Lemma* in another turn of phraſe, wherein the words *data quavis differentia* [*any difference that is or ſhall be given*] would have been improper. This author does not here diſtinguiſh between the expounding the ſenſe of a ſingle phraſe and repreſenting

ſenting the meaning of the whole ſentence, in which that phraſe is uſed.

§. 10. Any given difference is not in Mr. *Robins*'s opinion conſonant to any difference that can be aſſigned, though it be not difficult to make uſe of either of theſe phraſes in expreſſing the ſame thing, provided the reſt of the expreſſion be rightly accommodated to it.

Sect. VIII. §. 1, 2. Mr. *Robins* has never differed in the ſenſe, he has aſcribed to this *Lemma*, though he thought himſelf to apprehend its meaning ſo truly, as to be in no fear of expreſſing that meaning by different forms of words.

§. 3—9. To what purpoſe are theſe four ſuppoſitions here again repeated? Has not *Philalethes* yet diſcovered, that this interpretation of the *Lemma* makes it an erroneous propoſition? For two quantities may conſtantly tend to equality during ſome finite ſpace of time, and before the end of that time come nearer together than to have any difference, which ſhall be given; and yet at the end of that time have ſtill a real difference. For however ſmall the difference be, which is given; from the quantities attaining to that difference before the end of the time named, it will only follow, that at the end of this time they will have a leſs difference.

But *Philalethes* perhaps will ſtill inſiſt upon the words *any given difference* being ſynonimous to *any aſſignable difference*. And then two quantities are ſuppoſed conſtantly to tend to equality during ſome finite ſpace of time, and yet before the end of that time to come nearer together than to have any aſſignable difference. But thus the quantities will become equal before the end of that time, the whole of which is ſuppoſed to be taken up in their approach. [See *Republick of Letters* for *January*

nuary laſt at the bottom of pag. 82, and beginning of pag. 83.] As ſoon as quantities come to have no aſſignable difference, they are actually equal. By this interpretation therefore the terms of the *Lemma* are rendered inconſiſtent with themſelves.

§. 10—12. Does not *Philalethes* know, that the words here quoted from Mr. *Robins*'s Book were intended to expreſs only a part of Sir *Iſaac Newton*'s method, which relates to quantities approaching each other, and that the other part relating to ratios is expreſſed ſoon after in another paragraph? and has not Mr. *Robins* in his laſt paper united both theſe together? Why then does *Philalethes* repreſent Mr. *Robins*'s firſt deſcription of Sir *Iſaac Newton*'s method thus imperfect?

§. 13. None of the ſuppoſitions really contained in this *Lemma* are omitted by Mr. *Robins* in the paſſage here cited. For ſuppoſe the quantities are approaching to all eternity, they certainly then approach during any finite ſpace of time; and ſince the varying quantity or ratio is ſuppoſed capable of being made in its approach to the other to differ from it by leſs than any quantity, how minute ſoever, that can be aſſigned; certainly ſome finite ſpace of time may be always aſſigned, before the end of which the quantities or ratios may come nearer than by any difference whatever, that ſhall be propoſed. The concluſion of the *Lemma*, that the quantities become *ultimo æquales*, is not omitted; but Mr. *Robins* conceives truly interpreted; and *Philalethes* has no right to aſſert the contrary, till he has proved, that Sir *Iſaac Newton* deſigned no quantities or ratios to be comprehended within the Senſe of this *Lemma*, which do not become actually equal. This *Philalethes* has not yet proved, and Mr. *Robins* believes, never

ver will. That the quantities or ratios are to become equal at the end of the finite time mentioned in this *Lemma*, Mr. *Robins* apprehends is not the ſenſe of the *Lemma*.

§. 14. This ſecond interpretation was not given to ſupply the defects of the firſt, but to ſhew the truth of the firſt by expreſſing the ſame ſenſe in words nearer to thoſe of Sir *Iſaac Newton*.

§. 15. Here I conceive no ſuppoſition of Sir *Iſaac Newton* to be omitted, but that Mr. *Robins* has ſpoken of the finite time here mentioned in the ſenſe of Sir *Iſaac Newton*, although by a different rangement of the words it does not ſtand in the ſame place of both ſentences. Mr. *Robins* has indeed here omitted, and always intends to omit the ſecond ſuppoſition aſcribed by *Philalethes* to Sir *Iſaac Newton*, that the approach of the quantities or ratios mentioned is always to be conſidered within the limits of ſome finite ſpace of time; becauſe, he thinks, he has proved Sir *Iſaac Newton* to mean otherwiſe.

§. 16. Mr. *Robins* does not adviſe to read the demonſtration of this *Lemma* before the *Lemma* itſelf, in order to underſtand the meaning of the expreſſions uſed in it; that would have been to adviſe to ſeek after the meaning of words, before the words themſelves were read.

§. 17, 18. I am not of opinion, that the anſwer given was either full or concluſive; nor has Mr. *Robins* to evade it ever altered the ſenſe of the remark here quoted.

§. 19—22. Here Mr. *Robins* did not intend any third interpretation of this *Lemma*, and in order to repreſent what is quoted in §. 20, as ſuch, the words in §. 21 are falſly cited. For the words *the latter ſenſe* are turned into the word *this*. Thus Mr. *Robins* is made to call the preceeding words

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an interpretation of the *Lemma*, whereas *the latter ſenſe*, he ſpeaks of, is the interpretation, he aſſigns to the words, *given difference*, in oppoſition to the ſenſe they muſt have according to *Philethes*'s interpretation of that *Lemma*. The words cited in the 20th §. are not an interpretation of the *Lemma*, but an account how certain quantities may be brought under it, which *Philalethes* denies to be ſubjects of it.

§. 23. This is begging the queſtion, and proceeds upon the ſuppoſition, that *Philalethes* underſtands the true meaning of the words *ultimo æquales*, which Mr. *Robins* denies.

§. 24—26. Mr. *Robins* does not refer to the demonſtration of this *Lemma* for the ſenſe of the words *any given difference*, but expreſly fixes the ſenſe of theſe words from their genuine uſe in geometry. Mr. *Robins* refers to the demonſtration of this *Lemma* only for diſcovering the ſenſe of the phraſe *fiunt ultimo æquales*.

§. 27, 28. The ſuppoſition, that D is the laſt difference, is equally erroneous, whether the quantities are ſtill approaching, when they have this difference, which is ſuppoſed their laſt, or whether they have ceaſed to approach, but are already come nearer than by that difference.

§. 29. If *Philalethes* cannot underſtand Mr. *Robins*'s meaning, here called the fourth interpretation of the *Lemma* in queſtion, it is to be hoped his unprejudiced readers can. Suppoſe Mr. *Robins* means, that ſome of theſe quantities may perpetually and to all eternity have ſome difference, and yet no difference, that can be called their laſt; to what purpoſe is any queſtion here made about the meaning of the words *fiunt ultimo æquales*; for Mr. *Robins* has ſufficiently declared, what he thinks Sir *Iſaac Newton* meant by them,

them, and believes he has proved, that conſiſtent with truth Sir *Iſaac Newton* could not mean otherwiſe. And Mr. *Robins* ſuppoſes the ſenſe, he has aſcribed to them, not to be the plain and ordinary meaning of thoſe words, but a new ſenſe Sir *Iſaac Newton* thought fit to put upon them. Nay farther, it is not difficult to aſſign a very probable reaſon, which led Sir *Iſaac Newton* to the uſe of this expreſſion; for before him it had not been unuſual for geometers to ſpeak of the laſt ſums of infinite progreſſions, which is an expreſſion ſomething ſimilar to this. Surely here no one will pretend, that an infinite number of terms can, in ſtrict propriety of ſpeech, and without a figure, be ſaid to be capable of being actually ſummed up and added together.

§. 30. This paragraph depends entirely upon the uſe made of the finite time mentioned in this *Lemma*, and if Mr. *Robins* agreed in opinion with *Philalethes* relating hereto, the whole queſtion would be given up, and this paragraph unneceſſary; but, while he does not agree to it, this paragraph contributes nothing to the deciding the point in debate.

§. 31. In anſwer to the demand what is to be underſtood by the word *perpetually*, ſuppoſe it were anſwered, that it means all the time, that the approach of the quantities or ratios in queſtion is under conſideration.

§. 33. If it appears ſo ſurprizing to *Philalethes*, that one perſon ſhould in his opinion give four diſagreeing interpretations to the ſame thing, why may it not appear as ſtrange to Mr. *Robins*, that another perſon ſhould apprehend in four different ſenſes, what he thinks very evidently unite in the ſame?

Sect. IX. §. 1, 2. When in any phraſes common words are uſed in a new ſenſe, thoſe words may properly be called new terms; for though the ſound is the ſame, the ſenſe is different.

§. 3, 4. This is the only purpoſe, for which Mr. *Robins* refers to the demonſtration of this *Lemma* for the true underſtanding of it. And Mr. *Robins* thinks, that the method taken here by Sir *Iſaac Newton* is conſiſtent with his being a good writer, and has as much right to hold that opinion, as *Philalethes* has to be poſitive of the contrary. For *Euclide*, whom Mr. *Robins* eſteems as one of the correcteſt writers, that ever was, appears to have taken the like method with reſpect to the phraſe of compound proportion. For the definition found in the preſent copies of the Elements is neither alluded to by *Euclide*, when he firſt makes uſe of that term, nor is it well adapted to explain the ſenſe, in which it is uſed. But the ſenſe of the phraſe appears very evidently from his firſt uſe of it.

§. 5. How will *Philalethes* prove, that Mr. *Robins* was ever of any other opinion, than that Sir *Iſaac Newton*'s demonſtration is applicable to ſuch quantities, as at laſt become actually equal, as well as to quantities, which only approach without limit to the ratio of equality?

§. 6—8. Mr. *Robins* has indeed aſſerted, that ſome of theſe ſubjects do become actually equal. But ſuppoſe it were agreed, that, when quantities are capable of an actual equality, Sir *Iſaac Newton*'s demonſtration will prove it, what conceſſion is that to *Philalethes?* For whatever quantities *Philalethes* has named as becoming actually equal, which Mr. *Robins* has once denied to do ſo, he ſtill continues to deny as ſtrongly as ever. Suppoſe Mr. *Robins* ſhould allow, that *Philalethes*'s inter-

interpretation of the *Lemma* would agree with ſome of the quantities comprehended under it, is that a confeſſion, that *Philalethes* has expreſſed Sir *Iſaac Newton*'s true meaning, while Mr. *Robins* is of opinion, that his interpretation will exclude other quantities neceſſary to be brought under it? Surely that is declaring, that the interpretation of *Philalethes* is erroneous; becauſe the true interpretation ought equally to comprehend both.

§. 9. Mr. *Robins* perceived, that *Philalethes* miſtook his meaning in the firſt of the paſſages here cited; and therefore added the words, *neceſſarily implied*, for his information. What induced *Philalethes* to produce this latter quotation as if aſſerted of all quantities relating to this *Lemma*, when it only concerns a particular caſe, the parallelograms inſcribed and circumſcribed to a curve in the ſecond *Lemma?*

Sect. X. §. 1. Where has Mr. *Robins* admitted, that any quantities whatever will become actually equal at the end of the finite time referred to in the *Lemma?* If he has not, how has he allowed of *Philalethes*'s interpretation? What does *Philalethes* then mean by fancying, that Mr. *Robins* has at any time allowed Sir *Iſaac Newton*'s meaning to be the ſame with that of *Philalethes?* Mr. *Robins* ſays, that the *Lemma* in Sir *Iſaac Newton*'s ſenſe of it did not neceſſarily imply, that the quantities compared in it ſhould become actually equal, becauſe in fact many of them cannot; nay that Sir *Iſaac Newton* neither demonſtrated the actual equality of all quantities capable of being brought under this *Lemma*, nor that he intended ſo to do. See *Repub. of Lett.* for *April* laſt, p. 309.

§. 3, 4. Since *Philalethes* does not perceive, how the demonſtration of this *Lemma* can be applied to the caſe mentioned of the hyperbola, I

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ſhall now ſhew it at large. Here the hypotheſis aſſumed is a property of the hyperbola commonly known, That the curve continually approaches its aſymptote, as it is farther extended, ſo that by removing any ordinate farther and farther from the vertex, it will approach nearer and nearer in magnitude to the ſame continued to the aſymptote, without limit. Now to prove the ultimate equality of theſe two lines, let us ſay thus. If you deny it, let them be ultimately unequal, and let their laſt difference be D; therefore they cannot approach to equality nearer than by the given difference D, contrary to the hypotheſis. Thus is the demonſtration of this *Lemma* without changing a ſingle word applied to the preſent caſe.

§. 5—9. The reaſons here mentioned are not unanſwerable; one of them is this, *The demonſtration of this* Lemma *cannot poſſibly be applied to this caſe; ſince theſe two variable quantities can have no laſt magnitude, and conſequently cannot be ſuppoſed to have any laſt difference.* This argument of *Philalethes* depends upon this miſtake, that the difference propoſed as the laſt is a difference, which the quantities are ſuppoſed to have after the time of their approach is over. But there is no ground for ſuch a ſuppoſition from the words of Sir *Iſaac Newton*'s demonſtration. See the Remark on §. 28. of Sect. VIII.

The argument produced in *January* by interpreting the time mentioned in this *Lemma* to be one definite portion of time, and ſuppoſing the quantities to become equal at the end of that time, renders the *Lemma*, as we have above obſerved, a falſe propoſition.

§. 10. It never was any matter of doubt with Mr. *Robins*, whether the limitation of the finite time mentioned in this *Lemma* to ſome one definite

nite portion of time be neceſſary to the purpoſe of interpreting Sir *Iſaac Newton*'s meaning; but Mr. *Robins* declined in this place diſcuſſing that point, becauſe he could confute the objection under conſideration without it: for he ſhewed, how to put this caſe of the hyperbola ſo as to comprehend the motion propoſed in it within a definite ſpace of time. What does *Philalethes* mean in this next paragraph by ſaying, Mr. *Robins* has ſhewn this pretended method to be abſurd, fallacious and inconcluſive? Mr. *Robins* has only ſhewn, that the concluſion, which would be drawn after the addition of this circumſtance from *Philalethes*'s interpretation of this *Lemma*, is abſurd, fallacious and inconcluſive.

Sect. XI. How was it poſſible for *Philalethes* to aſſert, that Mr. *Robins* has not even offered to ſhew, that any quantities or ratios incapable of an actual equality are compared in this *Lemma*, when it is expreſsly endeavoured to be proved at pag. 315 of the *Republick of Letters* for *April* laſt, that this is abſolutely the caſe in all vaniſhing quantities, and that by the moſt natural interpretation of Sir *Iſaac Newton*'s ſecond and third *Lemmas*, thoſe *Lemmas* apply quantities to this firſt, which are incapable of an actual equality.

But does *Philalethes* really know, what thoſe quantities are which Mr. *Robins* acknowledges to be the ſubjects of this *Lemma*, and capable of an actual equality? When *Philalethes* ſhall name them, Mr. *Robins* will ſhew, that this *Lemma* is ſo far from being confined to that caſe, that the equality of all thoſe quantities may be proved by another method much more natural than by referring them to this *Lemma*.

The demonſtration of this *Lemma* is by the indirect form called *deductio ad abſurdum*. Now there

there are two kinds of quantities, which may be brought under this *Lemma:* one ſort are capable of having their ultimate equality proved alſo by the direct form of demonſtration; and theſe quantities do become actually equal: to the other ſubjects of this *Lemma* the direct form of demonſtration cannot be applied; theſe are the ſubjects, for the ſake of which this *Lemma* was principally writ; and theſe, in the opinion of Mr. *Robins*, never can arrive at an actual equality.

Sect. XII. §. 1—3. Here is a very poſitive charge of falſly tranſlating and interpreting and unfairly quoting certain words of Sir *Iſaac Newton.* It ſeems the words *pro æqualibus habeantur* ſhould have been rendered *let them be eſteemed equal.* But why is *habeantur* the Imperative mood? *Philalethes* was deceived by the preceeding words, which direct certain conſtructions; but here, where a conſequence is concluded from thoſe conſtructions, the Potential mood is required, in which, to uſe the learned *Linacer*'s words, *indicatio eſt potentiæ, debetive, aliquando voluntatis.* How then is, *are to be eſteemed equal*, a falſe tranſlation? Is not that *pro æqualibus haberi debent?* Juſt as *non expectes, ut ſtatim gratias agat, qui ſanatur invitus*, is expounded by this great Grammarian *non debes expectare.* See fol. 10. of his book *De Emendata ſtructura Latini Sermonis, Ed. R. Steph.* 1527.

I have quoted *Linacer*, becauſe he was the firſt who gave the name of potential to this mood, when it bears any of the three ſignifications here mentioned: but this form of the verb having all theſe ſenſes is a point agreed among Grammarians. See *Alvarez* and *Voſſius.*

The charge of quoting unfairly is wholly grounded upon certain words not being produced,

ced, which are nothing to the purpoſe: for it was the form of the expreſſion only, which Mr. *Robins* had here in view. Does *Philalethes* take it for granted, that Mr. *Robins* had ſo much as any ſhadow of ſuſpicion, that theſe ſubſequent words were anywiſe inconſiſtent with the ſenſe he has put upon the other? Surely *Philalethes* does not yet comprehend diſtinctly the point in diſpute between Mr. *Robins* and himſelf: elſe he would ſcarce have been guilty of ſo manifeſt a begging the queſtion, as is contained in theſe words, *When they come to vaniſh, they will arrive at the ratio of an abſolute equality.* Knows he not, that Mr. *Robins* allows the ratio called the ultimate of the vaniſhing quantities D F, *df* to be that of equality; though he will challenge *Philalethes* to ſhew, that it is conſonant either to the geometry of the ancients, or to truth, that theſe lines can be ever equal. If P S is greater than *p s*, D F muſt always be greater than *df*.

§. 4. Mr. *Robins* does not ſee, that of the two expreſſions, *ultimo in ratione æqualitatis* and *ultimo æquales*, the one will admit of a laxer interpretation any more than the other; for Mr. *Robins* has ever contended, that they muſt both have ſuch a lax ſenſe, when applied to quantities, that cannot actually exiſt under the condition of equality. What is here aſſerted concerning Mr. *Robins* ought to have been poſtponed, till the aſſertion be attempted to be proved.

§. 5. The ſecond *Lemma* was not produced in the account of Mr. *Robins*'s Diſcourſe as an example of the whole of the aſſertion juſt quoted from him, but only as an inſtance of one part of it; and therefore, although *Philalethes* had as manifeſtly ſhewn, as he here groundleſly preſumes, that *Lemma* to be an example quite contrary to

Mr. *Robins*'s purpoſe, it would not be an entire confutation of Mr. *Robins*'s aſſertion; nay it would be no confutation at all; for if Sir *Iſaac Newton* has applied equality to vaniſhing quantities, Mr. *Robins* has ſtill ſhewn the truth of his aſſertion.

§. 6. Surely *Philalethes* wrote this paragraph, before he had read Mr. *Robins*'s Diſſertation through; becauſe what he charges him with taking no notice of, he has diſcourſed of very largely at pag. 316. Mr. *Robins* thinks he had at firſt ſufficiently conſidered, what was ſaid by *Philalethes* in *November* laſt. *Philalethes* is deſired to explain, what he means here by ſaying Mr. *Robins* has made too free with Sir *Iſaac Newton*'s name; becauſe, I believe, Mr. *Robins* has ſuch a veneration for Sir *Iſaac Newton*'s memory, that he would be greatly grieved, that the publick ſhould ſuſpect him of treating him with the leaſt diſreſpect.

Sect. XIII. §. 1—11. *Philalethes* might eaſily have extricated himſelf out of all his confuſion by attending to one expreſſion of Mr. *Robins*'s original book, page 64. where it is obſerved, that by the form of a curve its tangent may meet the curve again in ſome other point: and at ſuch point the quantities, by whoſe ultimate ratio the ſituation of the tangent is determined, will bear that ratio, which is called their ultimate. And ſince quantities, whoſe ultimate ratio is ſought, may often have ſuch a relation to each other, as accidentally in particular magnitudes of them to bear the ſame ratio with that, which is called their ultimate; for that reaſon Mr. *Robins*, where he ſpeaks of vaniſhing quantities in general, has often taken care to turn his expreſſion ſo, as not to exclude this accident; but whenever he ſpeaks of vaniſhing

vaniſhing quantities not ſubject to this caſualty, he then always ſpeaks poſitively, that thoſe quantities never can be the ſubject of that ratio, which is called their ultimate. This is the caſe of the *Quadrilaterals* here mentioned in the third paragraph. However this reſtriction is of ſo little conſequence, that ſometimes perhaps it may not be ſo expreſly attended to. Mr. *Robins*'s preſent opinion concerning vaniſhing quantities is the ſame, it always was, and in ſhort is this; That there are no quantities at the inſtant of evaneſcence actually ſubſiſting to be the ſubjects of the ultimate proportion of the vaniſhing quantities; and this was Sir *Iſaac Newton*'s opinion, as appears from this expreſſion of his, there are *rationes primæ quantitatum naſcentium*, but not *quantitates primæ naſcentes*. Farther the definition of *Philalethes* of a naſcent increment being *an increment juſt beginning to exiſt from nothing, or juſt beginning to be generated, but not yet arrived at any aſſignable magnitude, how ſmall ſoever*, conveys to Mr. *Robins* no idea, nor does he believe to any one elſe, however any perſon through an indiſtinctneſs of conception may deceive himſelf. For ſurely this is a deſcription of infinitely little quantities, of which Sir *Iſaac Newton* ſays we have no idea.

§. 11. Mr. *Robins* had no occaſion to paſs any cenſure upon the paſſage here referred to. He never imagined, that *Philalethes* had not learnt from Sir *Iſaac Newton*, how to aſſign the ultimate proportion of vaniſhing quantities truly; his only objection to *Philalethes* is his giving a wrong and unintelligible notion of theſe quantities. This is an endeavour to divert the queſtion from the real point in diſpute.

Sect. XIV. §. 1, 2. Mr. *Robins* approves of the word *endleſly*, and is ſo far from acknowledging any miſtake it led him into, that on the contrary *Philalethes* has himſelf erred by imagining the words *in infinitum* have any other ſignification.

§. 3—6. *Philalethes* has given two ſenſes to this word *endleſly*, but neither of them will lead into error, but are both applicable to the preſent caſe. Nay the firſt ſenſe neceſſarily implies the ſecond; for if the number of theſe parallelograms is to be increaſed ſtill more and more without limit, it is certainly impoſſible for the imagination in any finite time to purſue to the end, what has no limit.

§. 7. Before *Philalethes* charges this interpretation as *petitio principii*, he ought to ſhew, that the words are capable of any other ſenſe, which I apprehend, it is impoſſible for him to do.

§. 8. Mr. *Robins*, I believe, is not aſhamed of his skill in ſelecting thoſe expreſſions, which he thinks contribute moſt readily to prove his opinion. Mr. *Robins* apprehends *Philalethes* in this paragraph to be chargeable with *petitio principii*; for that Sir *Iſaac Newton* by diminiſhing a quantity *in infinitum* is always to be underſtood, that he requires you to purſue that diminution in the imagination till the quantity is actually vaniſhed, Mr. *Robins* has as much liberty to deny, as *Philalethes* has to aſſert. And that Sir *Iſaac Newton* did not intend this in the propoſition in queſtion, he is well perſuaded, becauſe it is impoſſible.

§. 9. There is certainly no other harm in uſing improper expreſſions, when the writer is careful to explain his meaning, than in being guilty of a needleſs obſcurity.

§. 10.

§. 10. The fear Mr. *Robins* has of borrowing any terms from the language of indiviſibles ariſes from this, that they are abſolutely unneceſſary, and he believes, that Sir *Iſaac Newton*'s uſe of ſuch terms has been the whole occaſion of the miſunderſtanding his doctrine; for had no ſuch terms been uſed, whoever was capable of reading him, he thinks, muſt have come nearer to his true ſenſe.

Sect. XV. §. 1, 2. Mr. *Robins* thinks himſelf directed by the words of Sir *Iſaac Newton* to make the ſubdiviſion in the manner here propoſed. *Philalethes* in not underſtanding this place, confirms Mr. *Robins* in his opinion, how needful a knowledge in the ancients is to qualify a perſon for underſtanding either Sir *Iſaac Newton* or himſelf. But however *Philalethes* has endeavoured to ſhew his knowledge in the ancients by quoting two propoſitions from *Euclide*'s *Elements*; one of which teaches how to divide a line in the ſame proportion as ſome other line is divided, the other ſhews that by taking from any quantity more than half, and from the remainder more than half continually, the reſidue may be reduced within any degree of ſmallneſs. How much theſe propoſitions are to the purpoſe of inſcribing and circumſcribing parallelograms to a curvilinear ſpace, let *Philalethes* ſhew. When *Philalethes* has gone farther in the ancients than the *Elements of Euclide*, he will be better able to comprehend Mr. *Robins*'s meaning, and judge upon the point in queſtion.

§. 4. The motion here propoſed does not divide the baſe of the curve, as is required; at the end of half an hour the baſe will indeed be divided into two equal parts; at the end of three quarters of an hour one of thoſe parts will again be divided

into two halves; at the end of ⅛ of an hour one of theſe will again be biſected; but is this dividing the whole line continually into equal parts?

§. 5. Mr. *Robins* would perform nothing in relation to this third, nor yet the ſecond *Lemma* from the propoſitions above cited from *Euclide*, but ſuch ſubdiviſions as are required in both, he has performed in pag. 60 of his firſt *Diſcourſe*; for the baſe A D of the curvilinear ſpace A D B is divided into equal parts, and the baſe E H of the figure E F G H is divided unequally.

§. 6. After what has been writ on the fourth paragraph, I believe *Philalethes* will not deſire the queſtion here propoſed to be anſwered.

§. 7. I believe *Philalethes* will not repeat again this heap of aſſertions without proof, till he has found out ſome other motion, than he has yet been able to contrive, whereby to ſupport them; for, if there be no other method of inſcribing and circumſcribing the parallelograms, as Sir *Iſaac Newton* requires, but by ſubdividing continually the baſe of the curve; it is a wrong aſſertion, that Sir *Iſaac Newton* ſays not one word of continual diviſion or ſubdiviſion into parts of the baſe of the figure.

Sect. XVI. §. 1, 2. Beſides what has already been ſaid upon this pretended diviſion of the baſe by motion, it is farther neceſſary for *Philalethes*, in order to perform by continued motion what Sir *Iſaac Newton* requires, not only to divide the baſe of the curve, but form the parallelograms alſo by ſuch continued motion. And what motion will he contrive for this purpoſe, which ſhall not be perplexed or even confuſed?

Here let *Philalethes* be ask'd, where is to be found *Euclide*'s method of dividing the baſe of a curve

curve either to meaſure that curve, or to compare it with another curve?

§. 3. I anſwer, that the hour itſelf can certainly be purſued by the imagination to the end of it, but not the innumerable ſubdiviſions, which *Philalethes* pretends to have made by his motion. Perhaps I may be eaſieſt underſtood by comparing the preſent point with the old argument againſt motion from *Achilles* and the *Tortoiſe*. It is impoſſible to purſue in the imagination their motion by the means propoſed in that argument to the point of their meeting; becauſe the motion of each is deſcribed by the terms of an infinite progreſſion; but if we ſeek after the place of their meeting by the method propoſed in that argument, we muſt have recourſe to the doctrine of prime and ultimate ratios, and find the neareſt limit of each of thoſe progreſſions.

§. 4. Does *Philalethes* here mean, that it is not neceſſary in every geometrical demonſtration to form in the imagination a diſtinct conception of the ſubjects under conſideration? to do otherwiſe is not only contrary to the practice of the ancients, but to that of every other juſt reaſoner.

THUS we have gone through the *Conſiderations* of *Philalethes* publiſhed laſt month, paragraph by paragraph; and deſign to examine the reſt after the ſame manner, as ſoon as they ſhall be publiſhed. But to prevent the diſpute running into an unmeaſurable length, we ſhall afterwards reduce our *Remarks* into as narrow a compaſs as poſſible; and for that end intend to confine ourſelves to the diſcuſſion of one ſingle point, by which we apprehend the merits of the queſtion will in a manner be wholly decided. This is the examining whoſe interpretation of Sir *Iſaac Newton*'s firſt *Lemma* is agreeable to the nature of vaniſhing

quantities, as ſoon as *Philalethes* ſhall have rendred his interpretation a true and conſiſtent propoſition. But after this point is ſettled, we ſhall be ready to explain ourſelves upon any other, *Philalethes* ſhall deſire.

ARTI-

ARTICLE VI.

The REMAINDER *of the Paper begun in our laſt, entituled,* Conſiderations upon ſome paſſages of a *Diſſertation* concerning the Doctrine of *Fluxions,* publiſhed by Mr. *Robins* in the *Republick of Letters* for *April* laſt. By *Philalethes Cantabrigienſis.*

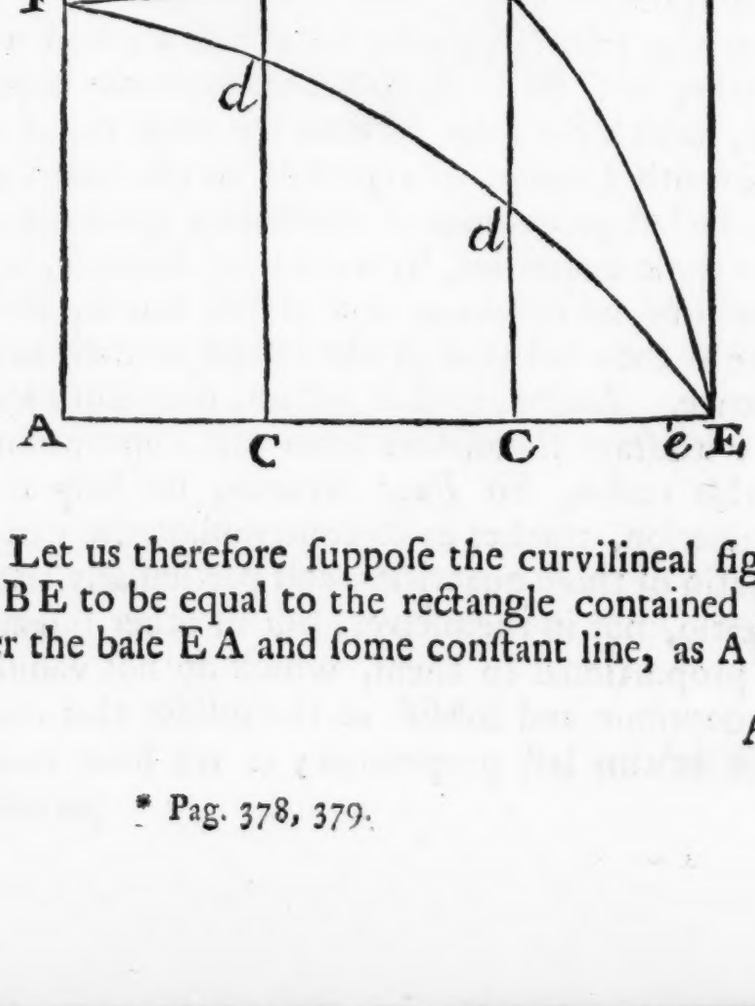
XVII. HOwever, ſince Mr. *Robins* is pleaſed to talk ſo much about *ſtraining our imagination for ſome involved and perplexed kind of motion,* let us ſee, if we cannot find ſome plain and eaſy way, of repreſenting to the imagination, that actual equality, at which the inſcribed and circumſcribed figures will arrive with each other, and with the curvilineal figure, at the expiration of the finite time.

For this purpoſe, perhaps the beſt method we can take, will be to imitate that judicious expedient, which Sir *Iſaac Newton* has made uſe of in the ſeventh *Lemma,* to repreſent to the imagination the laſt proportion of decreaſing quantities.

As thoſe quantities, by a conſtant decreaſe, are diminiſhed *ad infinitum,* and at laſt vaniſh, they arrive at their laſt ratio at the inſtant of their evaneſcence. And as, at that inſtant, they ſlip away and withdraw themſelves from our conception; for this reaſon, Sir *Iſaac Newton,* to help our imagination, teaches us to contemplate the variable ratio of thoſe quantities, and particularly their laſt ratio, not in themſelves, but in other quantities proportional to them, which do not vaniſh, but continue and ſubſiſt at the inſtant that they arrive at this laſt proportion; as we have more particu-

particularly obſerved in the *Republick of Letters* for *November* laſt.*

So here, as the inſcribed and circumſcribed figures, at the inſtant of their coincidence with the curvilineal figure, do, with reſpect to their rectilineal form, likewiſe ſlip away and withdraw themſelves from our conception; it will be of uſe to conſider their variable ratio, and particularly their laſt ratio, not in themſelves, but in other quantities, which undergo no alteration of their form, when they arrive at this laſt proportion.



Let us therefore ſuppoſe the curvilineal figure ABE to be equal to the rectangle contained under the baſe EA and ſome conſtant line, as AF.

And

* Pag. 378, 379.

And at the inſtant that the point, which we have † ſuppoſed to deſcribe the baſe E A in a finite time, arrives at any point as C, let the rectangle under the ſame baſe E A and ſome variable line, as C *d*, parallel to A B, be equal to the ſum of the inſcribed parallelograms, at that inſtant conceived as ſtanding upon C A the remaining undeſcribed part of the baſe, and upon as many other parts of the baſe E A, as can be taken equal to C A and adjoining to it, *i. e.* let the rectangle under E A and C *d* be equal to the inſcribed figure: And let E *d d* be the curve, which the point *d* continually touches or deſcribes.

In like manner, let the rectangle under the ſame baſe E A and ſome other variable line, as C D, alſo parallel to A B, be equal to the ſum of the circumſcribed parallelograms, at the ſame inſtant conceived to be ſtanding upon the ſame parts of the baſe as above, and upon E *e* the ſmaller remainder of the baſe, if any ſuch there be, adjoining to the point E, after the manner deſcribed in the *Republick of Letters* for *January* laſt; * *i. e.* let the rectangle under E A and C D be equal to the circumſcribed figure: And let G D D be the curve, which the point D continually touches or deſcribes.

Then, as the curvilineal, the inſcribed, and circumſcribed figures are reſpectively equal to theſe three rectangles E A × A F, E A × C *d*, E A × C D, having all the ſame common baſe E A; it is manifeſt that thoſe figures will be reſpectively proportional to the lines A F, C *d*, and C D; and may conſequently be always repreſented by theſe lines.

Now, ſince the two lines C *d*, C D, do

1. Con-

† Republick of Letters for *November*, p. 376.

* Page 90.

1. Conſtantly tend to equality with each other, and with the conſtant line A F,

2. During a finite time,

3. And approach within leſs than any given difference,

4. Before the end of that finite time; agreeably to the four ſuppoſitions of the firſt *Lemma*;

Theſe three lines will, by that *Lemma*, be equal to one another at the end of the finite time.

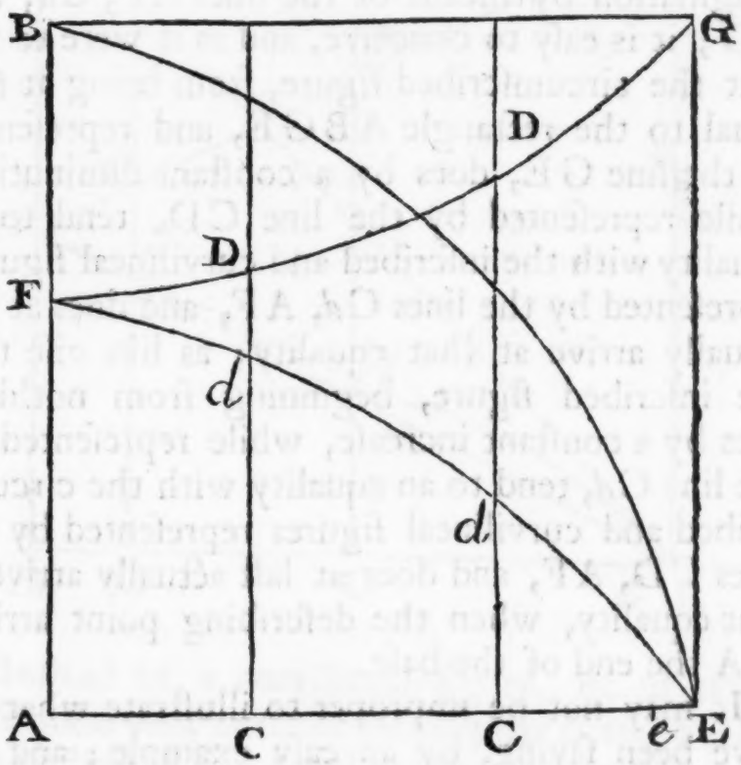
This we apprehend not only to be undeniably true, but *now* to be acknowledged even by Mr. *Robins*; ſince that Gentleman is at laſt brought to confeſs, that * whenever the "quantities compared in this *lemma* are capable of an actual equality, they muſt really become ſo;" and alſo that "† There is no ultimate ſum of theſe parallelograms, nor no ultimate figure compounded of them, diſtinct from the very curve itſelf." "This," he ſays, "Sir *Iſaac Newton*, in the corollaries annexed to the third *lemma* expreſsly declares." Here, it is manifeſt, that, by the *ultimate ſum of theſe parallelograms*, can only be underſtood the *ſumma ultima parallelogrammorum evaneſcentium*, which is ſaid *coincidere omni ex parte cum figura curvilinea*, in the firſt of thoſe corollaries; and that by the *ultimate figure compounded of them*, nothing elſe can be meant, than the *figuræ ultimæ*, of which it is ſaid, *non ſunt rectilineæ, ſed rectilinearum limites curvilinei*, in the fourth corollary; both which corollaries had been expreſsly || quoted and inſiſted on by me againſt Mr. *Robins*.

With this allowance therefore of Mr. *Robins*, we may ſafely aſſert, that, at the end of the finite time, the variable lines C *d*, C D, ſince they *are*

* Republick of Letters for *April*, p. 309.

† Ibid. p. 311. || Rep. of Let. for *Jan.* p. 84, 85.

are capable of an actual equality, muſt really become equal to each other, and to the conſtant line AF.



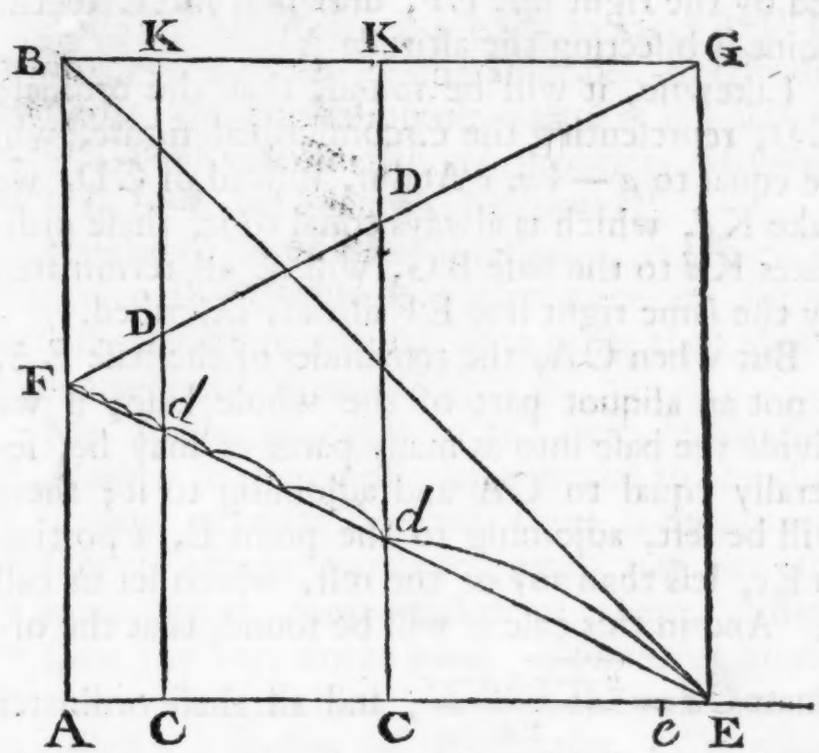
And, if there were occaſion, the ſame thing might alſo be proved after another manner. For, ſince the relation between C*d*, the ordinate to the curve E*dd*, and the abſciſs EC, may always be expreſſed by an equation to the curve E*dd*; and ſince the relation between CD, the ordinate to the curve GDD, and the ſame abſciſs EC, may always be expreſſed by an equation to the curve GDD: by means of thoſe two equations it may be made to appear, that, when the common abſciſs becomes equal to the whole baſe EA, theſe ordinates C*d*, CD, will be equal to each other, and to the conſtant line AF.

This

This being premiſed, and the proportion between the curvilineal, the inſcribed and circumſcribed figures, being always repreſented to the imagination by means of the lines AF, C*d*, and CD; it is eaſy to conceive, and as it were to ſee, that the circumſcribed figure, from being at firſt equal to the rectangle ABGE, and repreſented by the line GE, does by a conſtant diminution, while repreſented by the line CD, tend to an equality with the inſcribed and curvilineal figures, repreſented by the lines C*d*, AF, and does at laſt actually arrive at that equality; as likewiſe that the inſcribed figure, beginning from nothing, does by a conſtant increaſe, while repreſented by the line C*d*, tend to an equality with the circumſcribed and curvilineal figures repreſented by the lines CD, AF, and does at laſt actually arrive at that equality, when the deſcribing point arrives at A the end of the baſe.

It may not be improper to illuſtrate what we have been ſaying, by an eaſy example; and for my own and my reader's eaſe, I ſhall chuſe the ſimpleſt example I can think of.

Inſtead



Inſtead of a curvilineal figure, let us take a rectilineal one, *viz.* the rectangular triangle ABE, whoſe baſe AE is equal to the altitude AB. Let us imagine two figures, one inſcribed, and the other circumſcribed about this triangle, in the ſame manner as about the curvilineal figure in Sir *Iſaac Newton*'s ſecond *lemma:* And let theſe three figures be reſpectively repreſented, as before, by the conſtant line $AF = \frac{1}{2}AB$, and the variable lines, or ordinates, C*d*, CD.

Then, if EA, or AB be called *a*, and the part of the baſe, as EC, which at any inſtant of time is already deſcribed by the motion of the point C, be called *x*; it will eaſily be found, that, when CA the remainder of the baſe, is any aliquot part of the whole baſe, the ordinate C*d*, repreſenting the inſcribed figure, is equal to $\frac{1}{2}$ EC, or $\frac{1}{2}x$. Conſequently, all theſe ordinates will be termina-

ted

ted by the right line EF, drawn from E to the point F biſecting the altitude AB.

Likewiſe, it will be found, that the ordinate CD, repreſenting the circumſcribed figure, will be equal to $a - \frac{1}{2}x$. And if, inſtead of CD, we take K*d*, which is always equal to it, theſe ordinates K*d* to the baſe BG, will be all terminated by the ſame right line EF already deſcribed.

But when CA, the remainder of the baſe EA, is not an aliquot part of the whole baſe; if we divide the baſe into as many parts as may be, ſeverally equal to CA and adjoining to it; there will be left, adjoining to the point E, a portion as E*e*, leſs than any of the reſt, which let us call *r*. And in this caſe it will be found, that the ordinate $Cd = \frac{\overline{x+r} \times \overline{a-r}}{2a}$; and all theſe ordinates will be bounded by the continued curve E*dd*F, riſing a little above the right line EF, and touching it in every point, where an ordinate can be drawn to that curve from the end of any aliquot part of the baſe, lying between A and the middle of the baſe.

Alſo, it will be found, that the ordinate CD, or $Kd = a - \frac{\overline{x+r} \times \overline{a-r}}{2a}$; and all theſe ordinates K*d* will be bounded by the ſame curve E*dd*F.

Therefore, when $x = a$, and *r* vaniſhes, $Cd = \frac{1}{2}a$, and $Kd = \frac{1}{2}a$, i. e. C*d* and K*d* do then become equal to each other, and to the line AF. Conſequently the inſcribed and circumſcribed figures do then become equal to each other, and to the triangle ABE.

XVIII.

XVIII. Mr. *Robins* is pleaſed to ask,* "What "idea can we form of the inſcribed or circum- "ſcribed figure, to which we are at laſt actually "to arrive, which with any propriety of ſpeech "is to be ſtyled equal to the curve?"

I anſwer, The idea of the figure, at which we conceive the inſcribed or circumſcribed figures at laſt actually to arrive, is no other than that of the curvilineal figure itſelf.

Mr. *Robins* proceeds to tell us†, "Sir *Iſaac* "*Newton* in the corollaries annexed to the third "*Lemma* expreſly declares, there is no ultimate "ſum of theſe parallelograms, nor no ulti- "mate figure compounded of them, diſtinct "from the very curve itſelf. *Philalethes* himſelf "acknowledges this." And in the next line this is called a *conceſſion* of *Philalethes*. *Conceſſion!* To whom? Surely not to Mr. *Robins*. This Gentleman in the *Repub. of Letters* for *December*, *expreſly declares*, ‖ *The ſuppoſed laſt form of the inſcribed figure muſt eſſentially differ from the curve itſelf*. And again, ¶ *The curve is not the laſt form of the inſcribed figure.* ∴ *The coincidence does involve an impoſſibility.*

Would not therefore the paſſage before us ſhew a greater regard to truth, and be much more properly expreſſed, if it were to run in the following manner?

Sir *Iſaac Newton* in the corollaries annexed to the third *Lemma* expreſly declares, that * the laſt ſum of the evaneſcent parallelograms does perfectly coincide, *coincidit omni ex parte*, with the curvilineal figure; and † that theſe laſt figures are

* Rep. of Lett. for *Apr.* p. 310.
† Pag. 311. ‖ Pag. 446. ¶ Ibid.
∴ Ibid. p. 447. * Coroll. 1. † Corol. 4.

not rectilineal, but the curvilineal limits of rectilineal figures; and accordingly in the firſt propoſition of the ſecond ſection of the *Principia*, he tells us, the laſt perimeter of the triangles will be a curve line, by the fourth corollary of the third *Lemma*. *Philalethes* has always contended for this, has clearly proved and invincibly demonſtrated this, has inevitably forced and compelled me *B. R.* under the utmoſt unwillingneſs and reluctance, to confeſs and acknowledge this to be true.

Caſtigatque auditque dolos, ſubigitque fateri.

XIX. It ſeems, there is *a* ‖ *general complaint of the difficulties attending the ſtudy* of Sir *Iſaac Newton*'s doctrine, from the *brevity* and *conciſeneſs* with which it is delivered; *inſomuch that it often requires careful attention to diſcover the exact meaning, and full force of the expreſſions.*

A very ſad complaint truly! Sir *Iſaac Newton*'s doctrine of fluxions is not ſo eaſy to read as a Gazette, or a Play! If I will diſcover not only the meaning, but *the exact meaning*; not barely the force, but *the full force of* his *expreſſions*; it requires, nay, *it often requires* — What? Twenty years ſtudy by candle-light? No. But it requires attention. Nay more; it requires *careful attention.* I muſt confeſs, I was always of this opinion, and therefore have ſubmitted to this grievance of *careful attention*, and would adviſe other People to do ſo likewiſe.

XX. So much for the difficulty Sir *Iſaac Newton* puts upon thoſe who ſtudy his Doctrine: But it ſeems he is to blame likewiſe for the eaſe, which he gives ſome perſons in underſtanding him. "More * than once, to convey his intention the "*eaſier*

‖ Rep. of Lett. for April, p. 311. * Ibid.

" *eaſier* to thoſe, who had been accuſtomed to the " language of indiviſibles, he has introduced " ſome expreſſions analagous to the phraſeology " of that doctrine, when the brevity, he had " preſcribed to himſelf, occaſion'd his not giv- " ing expreſs notice of it."

Now, for my life, I cannot ſee that Sir *Iſaac* was at all to blame. In order to be underſtood, he uſed a language familiar to thoſe he wrote for. Mathematicians, when he firſt wrote, were univerſally accuſtomed to the method of indiviſibles, and the terms and expreſſions therein uſed; not only thoſe Mathematicians who approved that doctrine, but thoſe alſo, who upon examination rejected it, as inaccurate and erroneous. If therefore Sir *Iſaac Newton*, while *he addreſſes himſelf to thoſe, who had been accuſtomed to indiviſibles, accommodates himſelf to their language*; if, *to convey his intention the eaſier to them*, he *has introduced ſome expreſſions analagous to the phraſeology of that doctrine*, or even ſometimes uſed the very terms and expreſſions of that doctrine, I profeſs, I can ſee no harm in it. For, though he did not always *give expreſs notice of it*, yet he took care to caution his reader more than once, in what ſenſe he uſed thoſe terms or expreſſions; and thereby he was always underſtood, and never miſled any body that uſed *careful attention*. Though he never liked the doctrine of indiviſibles; yet he had skill enough to avoid the errors of that doctrine, without the ridiculous affectation of running perpetually out of his way, to avoid every term or expreſſion, that the Writers upon that ſubject had happened to make uſe of. He thought unintelligibleiſm, as great a fault as indiviſibleiſm.

XXI. Mr. *Robins* * will not allow of this. He gives us two expreſſions of Sir *Iſaac Newton*, where he *accommodates himſelf to the language* of indiviſibles, not in order *to convey his intention the eaſier to thoſe, who had been accuſtomed to that method*; (that were pardonable) but in one of them to ſpeak, as if there were a time when a thing ſhould happen, when his *words mean in reality no more, than that* the thing *will never* happen; and in the other to convey no intention at all, *no kind of meaning*.

——————*Dat inania verba,*
Dat ſine mente ſonum. Fumum, mirabile dictu,
Faucibus ingentem vomit, & caligine ſeſe
Involvit cæcâ, quales volitare figuras,
Morte obitâ, fama eſt ſolâ ſub nocte per umbram.

One of theſe expreſſions, *diminuuntur in infinitum*, is taken from the concluſion of his general *ſcholium* to the firſt ſection of the *Principia:* The other, *ſumma ultima parallelogrammorum evaneſcentium*, is uſed in the firſt corollary to his third *lemma*.

Now, for my part, I am ſo unhappy as to differ from Mr. *Robins*, both in thinking that theſe two expreſſions are taken from the doctrine of indiviſibles, or are any way *analogous to the phraſeology of that doctrine*; and likewiſe in ſuppoſing that they have *no kind of meaning*. Their meaning ſeems to me to be perfectly clear.

The former expreſſion, *diminuuntur in infinitum*, often occurs in Sir *Iſaac Newton*'s writings, particularly in the firſt ſection of his *Principia*, and ſignifies a † diminution *ſine limite*, till the quantities do *evaneſcere*, vaniſh into nothing. In the *Analyſis* written in or before the year 1669, we find

* Rep. of Lett. for April, p. 312. † Cogita diminuendas ſine limite. Scholium Sect. 1. Princip. ſub finem.

find the equivalent expreſſions, § *continuo diminuatur donec tandem evaneſcat*, and *diminuetur donec evaneſcat*, and *continuo decreſcit, donec tandem penitus evaneſcat*; as alſo ∴ *in infinitum diminui & evaneſcere, ſive eſſe nihil*, and ſometimes ſimply *evaneſcere*, which laſt word is frequently uſed either by itſelf, or after the words *coeuntibus punctis, redeat in locum priorem*, or ſome ſuch expreſſion, to determine its meaning, both in the firſt ſection of the *Principia*, and in the introduction to the *Quadratures*; to ſay nothing of the like expreſſions uſed in the account of the *Commercium Epiſtolicum*. So that, if this expreſſion have no meaning, or no clear meaning, the greater part of Sir *Iſaac Newton*'s writings muſt be unintelligible.

The ſecond expreſſion, *ſumma ultima parallelogrammorum evaneſcentium*, or *ſumma ultima*, as Mr. *Robins* prudently chuſes to quote it, he tells us, " in ſtrict propriety of ſpeech, has no kind of " meaning, for it is really infinite."

To which I muſt beg leave to reply, that his aſſertion is not true; as alſo that the proof he gives of his aſſertion is manifeſtly falſe; and farther, that, if what Mr. *Robins* gives for a proof, were ever ſo true, yet it would by no means prove his aſſertion.

The ſum of the parallelograms, *i. e.* the inſcribed or circumſcribed figures, during the approach of thoſe figures to one another and to the curvilineal figure, is an aggregate of a finite number of parallelograms finite in magnitude: And as theſe parallelograms diminiſh in breadth, and augment in number, *ſine limite*, their ſum conſtantly approaches to the curvilineal figure, and at laſt, at the end of the approach, degenerates into that figure, which is therefore called the *figura ultima*, or *ſumma ultima*, the figure, or ſum, to which

§ Demonſtr. reſol. æquat. aff. ∴ Præp. pro Reg. 1. dem.

they at laſt arrive. This is plainly Sir *Iſaac Newton*'s meaning.

Mr. *Robins*, in order to prove that the *ſumma ultima has no kind of meaning*, tells us *it is really infinite*. But the circumſcribed figure is finite at firſt, and conſtantly diminiſhes; and therefore can never grow *really infinite*. And the inſcribed figure, though it conſtantly increaſes, yet it never paſſes the magnitude of the curvilineal figure, which is finite: Conſequently, neither can the inſcribed figure become *really infinite*. Mr. *Robins*'s proof is therefore manifeſtly falſe.

And if this proof were true, if the *ſumma ultima* were *really infinite*; yet this would by no means prove Mr. *Robins*'s aſſertion, that *the ſumma ultima in ſtrict propriety of ſpeech, has no kind of meaning*; unleſs the words *really infinite, in ſtrict propriety of ſpeech*, have *no kind of meaning*. But this, ſurely, is what no body can ſurmiſe, that Mr. *Robins* himſelf, who has laboured ſo much *to preſerve propriety of expreſſion*, ſhould have uſed an expreſſion, which *in ſtrict propriety of ſpeech, has no kind of meaning*.

So far is that Gentleman from this fault, of having *no kind of meaning* in what he ſays, that he too often falls into the other extreme of having too much meaning, or rather too many meanings. For where Sir *Iſaac Newton* has only one meaning, Mr. *Robins* has ſometimes two or three; and what is ſtill a greater misfortune, of thoſe two or three meanings, there is none to be found that will anſwer his purpoſe. For inſtance, in the caſe before us, the *ultima ſumma* is ſaid to be infinite, and to give more force to the expreſſion, it is ſaid to be *really infinite*. But what is *the genuine meaning* of theſe words *really infinite?* Is it, that the *ſumma ultima* is infinite in magnitude? That it contains an infinite area? If ſo; we have already ſhown this to be manifeſtly falſe. Or

Or does he mean, that the *ſumma ultima*, tho' of a finite magnitude, contains an infinite number of parallelograms? But to ſay *the number* of parallelograms *becomes infinitely great, is the expreſs language of indiviſibles.* Beſides, if we may be allowed to ſay, the *ſumma ultima is really infinite*, becauſe the number of parallelograms may exceed any number that ſhall be propoſed; we may, with equal *propriety of ſpeech*, affirm, that a line of an inch long *is really infinite*, becauſe the number of parts into which it can be divided, may likewiſe exceed any number that ſhall be propoſed.

XXII. Mr. *Robins* goes on thus. "To aſſert "that any collection of theſe inſcribed or cir- "cumſcribed parallelograms can ever become "actually equal to the curve," (curvilineal figure rather in *ſtrict propriety of ſpeech*) "is cer- "tainly an impropriety of ſpeech; for equality "can properly ſubſiſt only between figures "diſtinct from each other."

And here, I could almoſt ſit down and deplore my own misfortune, in having to do with a Writer, who ſo often, in a ſingle ſentence, almoſt with one daſh of his pen, makes no conſcience of cutting me out work for ſeveral pages. But the task, however uneaſy, muſt be ſubmitted to, or I ſhould be unworthy the name I bear, of *Philalethes.*

I muſt therefore again obſerve, that Mr. *Robins*'s aſſertion in this ſentence is utterly untrue; that the proof he gives of that aſſertion, is manifeſtly falſe, even by his own confeſſion; and farther, that, if what he gives us for proof of his aſſertion, were true, yet it would be ſo far from proving his aſſertion, that it would prove the direct contrary of his aſſertion to be true.

Whether ſuch collection of parallelograms ever is, or *can ever become* actually equal to the curvilineal figure, is not now the queſtion. I have clearly expreſſed my ſentiment upon this point, though in other words than what Mr. *Robins* would here impute to me, in ſeveral places, particularly in the *Republick of Letters* for *November* laſt, p. 377. and in that for *January*, p. 87, 88. which paſſages remain as yet unanſwered. But let what I have there ſaid, be true, or falſe, yet, as this collection of parallelograms is a finite area, and the curvilineal figure is likewiſe a finite area, it can *certainly* be no *impropriety of ſpeech*, to aſſert that one is, much leſs that one *can ever become actually equal* to the other. Mr. *Robins*'s aſſertion is therefore untrue; let us ſee what we can make of his proof.

That proof is, *Equality can properly ſubſiſt only between figures diſtinct from each other.*

To which I muſt oppoſe the known axiom of *Euclid*, upon which is grounded the demonſtration of many propoſitions in his *Elements*, *Quæ mutuo ſibi congruunt, ſunt æqualia.*

And Mr. *Robins* himſelf, in other places, acknowledges the ſame thing. In his *Diſcourſe upon Fluxions*, p. 44. he tells us, "The primary "method of comparing together the magnitudes "of rectilinear ſpaces is by laying them one up-"on another." And preſently after he ſays, "This method cannot be applied in comparing "curvilinear ſpaces with rectilinear ones; becauſe "no part whatever of a curve line can be laid "upon a ſtreight line, ſo as wholly to coincide "with it."

Alſo, in the *Republick of Letters* for *October*, p. 256. this Gentleman tells us, "One (way of "comparing ſpaces) is by ſhewing them to con-

"ſiſt

" ſiſt of ſuch, as by impoſition on each other " will appear to occupy the ſame ſpace." And in the very ſame ſentence he gives us to underſtand, it is *indubitable, that thoſe things are equal, which have no difference.*

From all which I muſt conclude that, in Mr. *Robins*'s own opinion, when his judgment is not perverted by heat of controverſy, *equality can properly ſubſiſt between figures* not *diſtinct from each other*; unleſs any man will tell me, that figures, which are *laid one upon another*, which *wholly coincide* with one another, which *by impoſition on each other occupy the ſame ſpace*, are *diſtinct from each other*; unleſs any man will be hardy enough to deny, what Mr. *Robins* avers to be *indubitable, that thoſe things are equal, which have no difference.*

But let us for once allow, that Mr. *Robins*'s proof is true, that *equality can properly ſubſiſt only between figures diſtinct from each other*; yet it will be no proof of his aſſertion, but of the contrary. For ſince *any collection of theſe inſcribed or circumſcribed parallelograms* is diſtinct from the curvilineal figure, equality may *properly ſubſiſt* between them, one may *in ſtrict propriety of ſpeech* affirm, that they may become equal to one another.

The remainder of this paragraph of Mr. *Robins* does not, I think, concern either myſelf, or my honoured friend and new Ally, the *Author of the Analyſt*. It ſeems to be levelled againſt ſuch only, as are tainted with the damnable hereſy of indiviſibles, which is far from being our caſe.

XXIII. I therefore proceed to the queſtion put to me in page 313, to which I ſhall return an anſwer very clear and explicit.

" Does *Philalethes* here ſuppoſe the truth of " Sir *Iſaac Newton*'s demonſtrations to depend on " this

" this actual equality of the variable quantity and " its limit?" I do.

" He confeſſes our demonſtrations to be juſt, " which do not ſuppoſe this actual equality."

Yes, * *in the manner Mr.* Robins *defines, and treats of prime and ultimate ratios,* I allow his demonſtrations to be juſt without this actual equality. But Sir *Iſaac Newton* does not define and treat of prime and ultimate ratios, in the ſame manner with Mr. *Robins*; nor are Mr. *Robins*'s demonſtrations at all like Sir *Iſaac Newton*'s demonſtration.

XXIV. " He (*Philalethes*) alſo ſays, that *the* " *ſuppoſing this actual equality, ſeems greatly to ex-* " *ceed the method of the ancients in perſpicuity, as* " *well as in the conciſeneſs of the demonſtrations.*"

Here I muſt complain, that I am unfairly dealt with. The paſſage referred to, is to be found in the *Republick of Letters* for *November*, p. 373. where I firſt obſerve, that Sir *Iſaac Newton takes in the conſideration of a finite time*; ſecondly, that *thereby he greatly aſſiſts the imagination*; and I afterwards ſay, *On both theſe accounts*, (not, as Mr. *Robins* repreſents it, *by ſuppoſing this actual equality*) *this method ſeems greatly to exceed the method of the ancients, in perſpicuity as well as in the conciſeneſs of the demonſtrations.*

But we are told, " That this method ſhould " be more perſpicuous is impoſſible, the method " of the ancients being perfect in that reſpect."

Herein I greatly differ from Mr. *Robins*. A teleſcope compoſed of an hundred glaſſes will not be equally diaphanous with one compoſed of two; although every one of the hundred glaſſes, ſingly taken, be equally tranſparent with each of the two. Nor can a demonſtration, conſiſting of an hundred

* Rep. of Let. for *Jan.* p. 79.

hundred ſteps, be equally perſpicuous with one conſiſting of three only; though every one of the hundred ſteps, ſingly taken, be equally clear with each of the three.

Beſides, a direct demonſtration is certainly, *cæteris paribus*, more perſpicuous than an indirect one, which reduces the reader *ad abſurdum*. In the former, the Reader may ſee that every ſtep tends towards the concluſion: In the latter, he is generally as it were hoodwink'd and led quite away from the concluſion itſelf, till he comes to ſome abſurdity, which he muſt either acknowledge, or admit the concluſion to be true; as finding no poſſibility of avoiding either the one or the other. Now, as Sir *Iſaac Newton*'s firſt and fundamental *lemma*, though demonſtrated *ab abſurdo*, is yet a very ſhort and eaſy one, in which the imagination is led towards the concluſion by the conſideration of a finite time, even before one begins to conſider the demonſtration; and almoſt all his other demonſtrations are direct ones, and are likewiſe ſhort and clear, I may with reaſon ſay, that his method greatly exceeds the method of the ancients, where the demonſtrations are indirect, tedious and perplexed, in perſpicuity as well as conciſeneſs. Indeed, the perſpicuity of a demonſtration is always in proportion to its conciſeneſs, if the ſingle ſteps be clear.

XXV. But it is ſaid, "Certainly there are not "in their method, what *Philalethes* (though I "think without reaſon) inſinuated of this, any "demonſtrated truths, that muſt be owned, tho' "we do not perfectly ſee every ſtep, by which "the thing is brought about."

The paſſage of mine here alluded to, is not, as the Reader may imagine, and as Mr. *Robins* ſeems

to

to deſign he ſhould imagine, any way connected to, or ſo much as to be found in the ſame treatiſe with the paſſage that was laſt under conſideration, wherein I ſpoke of Sir *Iſaac Newton*'s method as exceeding the method of the ancients, in perſpicuity, as well as in the conciſeneſs of the demonſtrations. That was contained in the *Republick of Letters* for *November* laſt, and this is to be found in the *Republick of Letters* for *January* laſt, p. 87. and runs in theſe words.

"This equality therefore we are obliged to ac-
"knowledge, although we ſhould not be able,
"by ſtretch of imagination, to purſue theſe fi-
"gures, and, as it were, to keep them in ſight
"all the way, till the very point of time that
"they arrive at this equality. For a demonſtra-
"ted truth muſt be owned, though we do not
"perfectly ſee every ſtep, by which the thing is
"brought about."

It is eaſy to ſee, that the ſteps ſpoken of in this paſſage, are not the ſteps of the demonſtration, but the ſteps of the imagination; not the ſteps by which the thing is proved to be true, (that proof is ſuppoſed to be already over and finiſhed) but the ſteps by which the imagination can conceive, and, as it were, ſee the thing to be brought about.

Now, I ſay, there are in the method of the ancients many demonſtrated truths, that muſt be owned, though it be utterly impoſſible for the imagination to ſee how, or by what ſteps, the thing is brought about.

That the aggregate of three equal cones is equal to a cylinder of the ſame baſe and height, is a demonſtrated truth in the method of the ancients; and though it be very eaſy to ſee that the ſteps of the demonſtration are true and certain, yet it will not be eaſy for the imagination to ſee how, or by what

what ſteps, theſe three cones can be united to compoſe the cylinder.

Alſo, if one of theſe cones, keeping always the ſame baſe, be ſuppoſed gradually to increaſe in height, till its altitude become triple of the altitude of the cylinder; it is then a demonſtrated truth, that ſuch cone will be at laſt equal to the cylinder; and yet the imagination will not be able to perceive either this equality, or how it is brought about.

It is likewiſe a demonſtrated truth, that the 1000 external angles of a rectilineal figure of a thouſand ſides, whether thoſe ſides are all equal or any how unequal, will be equal to four right angles; and yet the imagination cannot be brought to ſee, how the thouſand angles compoſe the four, in any one of theſe caſes; much leſs in the endleſs variety of them that might be produced. Nor indeed can the imagination repreſent to itſelf a figure of 1000 ſides, ſo as to diſtinguiſh it from another of 1001.

The remainder of this paragraph and the next is chiefly taken up, with repreſenting what we have ſaid, about the motion of a point deſcribing a line, as ſomething *intricate*, *ſufficiently intricate*, *involved and perplexed*, *ſubtle*, *complex*, *not to ſay confuſed*, ſuch as *will bring us upon the very borders of indiviſibles*, *and* not only ſo, but will *render us perpetually obnoxious to the abſurdities of that doctrine*. Now Heaven preſerve all honeſt men from indiviſibles, whoſe *very borders* are ſo dangerous!

Nulli fas caſto ſceleratum inſiſtere limen.

One may travel, not only to the borders of *France*, or *Turky*, but through the heart of thoſe regions of ſlaves; nay one may reſide among them for years together, and yet not be rendered *perpetually obnoxious* to arbitrary power.

XXVI. In the four following pages I find nothing relating to me, except only that in p. 316 it is ſaid, Sir *Iſaac Newton* "draws a parallel be-"tween this caſe," (the caſe of vaniſhing quantities) "and the caſe of quantities ſuppoſed to aug-"ment without end." And a little after we are told, "*Philalethes* is very unwilling to allow this "intended for an exact parallel." Mr. *Robins* might have ſaid, *Philalethes* has clearly * proved, that this was not intended for an exact parallel.

"But, ſays Mr. *Robins*, Sir *Iſaac Newton* ex-"preſly affirms, his intention of introducing it "was to render more clear the thing, he had "been immediately ſpeaking of, that is, the na-"ture of the ultimate ratios of quantities decrea-"ſing without limit.

To this I reply, that although Sir *Iſaac*'s intention was to *render more clear* what went before, yet that will not prove this to be *an exact parallel* to what went before. Every illuſtration is not *an exact parallel*. It is ſufficient, if the illuſtration agrees with the thing to be illuſtrated, in ſome one particular, which needed to be rendred more clear. So here the quantities *ſuppoſed to augment without end*, agree with the quantities *decreaſing without limit*, in this one particular, that the *quantitates ultimæ* cannot be aſſigned in one caſe, any more than in the other: but ſurely the caſe of quantities increaſing without limit cannot be an exact parallel to the caſe of quantities decreaſing without limit. Beſides, I have † ſhown, that the ratio of the decreaſing quantities may arrive at its limit; but that the ratio of the increaſing quantities can never arrive at its limit: ſo that theſe two caſes are far from being exactly parallel.

Indeed,

* Repub. of Letters for *Nov.* p. 382, 383. † Ibid.

Indeed, Mr. *Robins* himſelf, preſently after, indirectly confeſſes this to be true. "It is true, "ſays he, the quantities in one caſe may be reduced abſolutely to nothing, but in the other can "never be extended to an infinite magnitude."

Now, when the decreaſing quantities are reduced abſolutely to nothing, I have clearly ſhown; not only in the laſt quoted place, but more particularly ſtill, and in a manner uncontroverted by Mr. *Robins*, in the *Republick of Letters* for *January* laſt, p. 76, 77. how the ratio of thoſe quantities will arrive at its limit. And if that be not ſufficient, the ſame thing may be proved from what that Gentleman himſelf lays down in this very page. For after telling us, that "theſe ra-"tios can never paſs, nor ever reach thoſe limits, "before the quantities are diminiſhed to nothing," he now acknowledges that *the quantities may be reduced abſolutely to nothing:* conſequently the ratio of thoſe quantities will arrive at its limit.

And as it is agreed on both ſides, that the ratio of increaſing quantities will never arrive at its limit, I may very ſafely affirm, that the two caſes are not exactly parallel.

I muſt not forget to obſerve, that what Sir *Iſaac* by this illuſtration was endeavouring to render more clear, was not, as Mr. *Robins expreſly affirms*, *the nature of the ultimate ratio's of quantities decreaſing without limiti* but only that, when quantities decreaſe without limit, the *quantitates ultimæ* cannot be aſſigned.

The reſt of what is ſaid in this and the two next pages, about thoſe decreaſing quantities, and their not being the *ſubjects* of the *ratio ultima*, as well as what is ſo often repeated in other places, that they cannot, or may not, or will not *ſubſiſt*, or *exiſt*, under that ratio, may ſerve to ſhew how well

well Mr. *Robins* can fight with his own ſhadow; but does not at all concern me, or my learned friend, the Author of the *Analyſt*. We hold no ſuch opinion.

XXVII. But in page 319 that Gentleman is again accuſed by Mr. *Robins* of committing a twofold error; of neither of which I can in my conſcience allow him to be guilty, in the manner thoſe errors are repreſented by Mr. *Robins*; and when rightly repreſented, if they are really errors, thoſe errors are much more juſtly to be imputed to me, than to him. Conſidering the bickerings we have had together, who could ever have expected, that excellent perſon ſhould have proved ſo great a Confeſſor in my cauſe? What could move Mr. *Robins* to uſe him thus ſeverely, on account of a controverſy, in which he was utterly unconcerned? Whatever his mathematical ſins have been, they were committed long before Mr. *Robins*'s *Diſcourſe* was written; and yet he was never once mentioned in that *Diſcourſe*. Beſides, thoſe ſins have been attoned for by ſo ſevere a penance, that, I profeſs, my bowels yearn for him, every time I think of it. Has Mr. *Robins* leſs compaſſion? I cannot think it. I am no ſtranger to this Gentleman's generoſity and good nature, which will not eaſily permit him to inſult over the unfortunate and unſucceſsful. What then can be the meaning of his being now, juſt now, ſo hardly treated? It cannot be ſure, that, with regard to certain points, it is thought eaſier to attack him, than to reply to me. For, though by this means the trouble of anſwering my arguments may be ſaved, yet will not that learned perſon be able to produce as good, or better arguments in vindication of his opinions?

Are

Are his abilities to be judged of by his making a bad defence, when unfortunately he happened to be engaged in a bad cauſe, any more than thoſe of Mr. *Robins*, from his making a worſe defence in the like caſe? Give the Author of the *Analyſt* the right ſide of the queſtion, and he may be turned looſe againſt any diſputant in *Europe*. Had this been the caſe, when he firſt attacked the *Britiſh* Mathematicians, it would by no means have been ſafe for Mr. *Robins*, or me, to have encountered him.

——*Non illi ſe quiſqnam impune tuliſſet*
Obvius armato. Stetimus tela aſpera contra,
Contulimuſq; manus: Experto credite, quantus
In clypeum adſurgat, quo turbine torqueat haſtam.

But as I know not whether he has at this time leiſure or inclination to defend himſelf, I ſhall, as I at firſt engaged, uſe my beſt endeavours to do it for him.

The firſt of theſe errors is ſaid to conſiſt " in " imagining, that the operation, which the in- " crements are by Sir *Iſaac Newton* ſuppoſed to " undergo in order to have their ultimate propor- " tion aſſigned, and which he deſcribes by the " verb *evaneſcant*, is confined to that point of " time only, at which the increments are actually " gone and aboliſhed."

And here, before either the *Author of the Analyſt*, or I *Philalethes*, can plead guilty, or not guilty, to this charge, it will be neceſſary to underſtand what the error is. In order whereto, I muſt beg leave to ask one or two queſtions.

1. What is meant by *the operation, which the increments are ſuppoſed to undergo?*

2. What is *that point of time only, at which the increments are actually gone and aboliſhed?*

1. If by this *operation* is meant a gradual diminution of the increments, ending in their total abolition or vaniſhing into nothing; I cannot find that Sir *Iſaac Newton* has ever *deſcribed* this *operation by the verb evaneſcant.* What he *deſcribes by the verb evaneſcant*, is only the end of that diminution, the vaniſhing into nothing; not the diminution itſelf, which precedes, and which ends in that vaniſhing. Though the diminution, when it is not mentioned, is ſometimes implied by the verb *evaneſcant*; yet it is never deſcribed by that verb.

Much leſs will it be found, that either the *Author of the Analyſt*, or myſelf, have ever confined ſuch a gradual diminution to one *point of time only.* We are not ſo weak, as not to know, that a gradual diminution requires a finite time.

And if, by this *operation*, is meant only the end of the gradual diminution of the increments, *i. e.* their total abolition, their vaniſhing into nothing, which Sir *Iſaac Newton deſcribes by the verb evaneſcant*; I ſhall then readily admit, that the *Author of the Analyſt*, or at leaſt, that I *Philalethes* have imagined, that this *operation* is confined to one *point of time only:* But it is to that point of time, at which the increments vaniſh into nothing; not to a point of time, *at which the increments are actually gone and aboliſhed.* For

2. It is eaſy to conceive, that the point of time at which the increments vaniſh, is one, determinate, only point of time: But I can have no conception of one *point of time only, at which the increments are actually gone and aboliſhed.* When once

once they are gone and aboliſhed, if we take any point of time at pleaſure, during any finite time how great ſoever, we may truly ſay, at this point of time the increments are actually gone and aboliſhed; but not at this *point of time only*. A little more accuracy in *ſtyle and diction* would do none of us any hurt; we all need it at leaſt as much as Sir *Iſaac Newton*. But it is eaſy to ſee, that through this cloud of expreſſion, though the *Author of the Analyſt* is only named, yet *Philalethes* is the perſon really aimed at. I proceed therefore to the real point, which Mr. *Robins* has here in view.

That by the word *evaneſcere* Sir *Iſaac Newton* underſtood, that decreaſing quantities did at laſt vaniſh into nothing, or ceaſe to exiſt, in an inſtant or point of time, was ſo clearly proved in the month of *November* * laſt, particularly by the words then quoted from him, *intelligendam eſſe rationem neque antequam evaneſcant, neque poſtea, ſed quacum evaneſcunt*, that I never expected to have heard more of this part of the diſpute, eſpecially after Mr. *Robins* had once dropt it in his reply to me in *December* following.

But as this point is now revived, together with ſeveral others, which had likewiſe been dropt, as well as many new ones *now* firſt brought into controverſy; I ſhall once more reſume the conſideration of it, and ſhall confirm my ſentiment by new authorities taken partly from Sir *Iſaac Newton*, and partly from his great interpreter Mr. *Robins*.

I aſſert then, That quantities, which, by a continual diminution, at laſt vaniſh into nothing and ceaſe to exiſt, are by Sir *Iſaac Newton* underſtood to vaniſh in an inſtant or point of time. Before

* Republick of Letters for *November*, p. 384, 385.

that inſtant, they are finite quantities, and after that inſtant they are nothing.

Likewiſe, quantities, which beginning from nothing, do by a continual increaſe, arrive at ſome finite magnitude, are by Sir *Iſaac Newton* underſtood to begin to exiſt, to ariſe out of nothing, *naſci*, at one inſtant or point of time. Before that inſtant they are nothing, and after that inſtant they are finite quantities.

This will plainly appear from the following expreſſions, in which Sir *Iſaac* ſpeaks of the proportion of naſcent or evaneſcent quantities, as one certain, determinate proportion, which conſequently is confined to one inſtant of time, at which the quantities begin to exiſt, or ceaſe to exiſt.

[a] *Rationem quacum evaneſcant.* [b] *Ratio quacum naſcuntur.* [c] *Ordinatæ ſunt ut arearum augmenta naſcentia.* [d] *Augmentum evaneſcens* Bb *erit ad augmentum evaneſcens* Ee, *ut* AB×FB *ad* AE×PE. [e] *Incrementum primum ſeu differentia prima cui naſcenti proportionalis eſt ejus fluxio prima.* [f] *Incrementum ſecundum, ſeu differentia ſecunda cui naſcenti proportionalis eſt ejus fluxio ſecunda.* [g] *Incrementum tertium, ſeu differentia tertia cui naſcenti fluxio tertia proportionalis eſt.*

In all theſe expreſſions the reader will obſerve, there is no ſuch word as *prima*, or *ultima*, *primò*, or *ultimò*, to be found; though in other expreſſions thoſe words are ſometimes added by way of *emphaſis*, which led Mr. *Robins* into the miſtake of thinking them always neceſſary.

Any one therefore of theſe paſſages, much more all put together, is ſufficient to ſhew, that, by *vaniſhing*, Sir *Iſaac Newton* does not mean the ſame

[a] Schol. ſect. 1. Princip. [b] Ibid. [c] Introd. ad Quadr. Curv. [d] Ibid. [e] Schol. Quadr. Curv. [f] Ibid. [g] Ibid.

ſame as *decreaſing*; that he does not call the *quantities vaniſhing from the time he firſt aſcribes to them a perpetual diminution*, nor conſequently ſuppoſes them to bear to one another an infinite number of different proportions, while they are *vaniſhing*; but that, having *decreaſed* by a continued motion, and with a variable ratio, they are underſtood to *vaniſh*, at the inſtant when they come to nothing or ceaſe to exiſt; at which inſtant their variable ratio arrives at and ends in that one determinate ratio, with which they are ſaid to *vaniſh*.[h]

This diſtinction between *decreaſing* and *vaniſhing* is farther confirmed by the following paſſages of Sir *Iſaac Newton*, which I have already quoted on another occaſion.

[i] *Continuo diminuatur donec tandem evaneſcat: Diminuetur donec evaneſcat. Continuo decreſcit, donec tandem penitus evaneſcat. In infinitum diminui & evaneſcere.* [k] *Minuetur in infinitum & ultimo evaneſcet.* Theſe plainly ſhew, that *vaniſhing* is only the end or concluſion of the conſtant decreaſe or diminution.

To all which I muſt beg leave to add a few more quotations of another kind, by which Sir *Iſaac Newton*'s intention is ſtill more manifeſtly to be diſcerned.

[l] *Punctis coeuntibus angulus evaneſcet.* [m] *Angulo evaneſcente coincident areæ.* [n] *Redeat jam* Pb *in locum ſuum priorem* PB *ut augmenta illa evaneſcant.*

So many inſtances of all theſe ſorts, but particularly of naſcent or evaneſcent quantitities having one only, determinate ratio, are to be found in the

[h] Rep. of Lett. for *Jan.* p. 76, 77.
[i] Analyſis per æquationes num. term. infinitas.
[k] Princip. ſect. 1. Lem. 6.
[l] Princip. ſect. 1. Lem. 7, & 8. [m] Ibid. Lem. 9.
[n] Introd. ad Quadr. Curv.

Principia, that no body can pretend to have read that book with underſtanding and *careful attention*, who has not obſerved them.

To theſe citations from Sir *Iſaac Newton* I muſt add one or two from Mr. *Robins*, which plainly ſhew this ſenſe of the word *vaniſhing* to be ſo common and natural, that this Gentleman himſelf cannot help falling into it, even while he is ſtrongly contending that the word is to be taken in another meaning.

In one place he ſays, * the line D F may be ſuppoſed to advance twards A E with an uninterrupted motion, till the quadrilaterals quite diſappear, or *vaniſh*. In another, he tells us, ‖ the quadrilaterals become vaniſhing quantities from the time, we firſt aſcribe to them this perpetual diminution; that is, from that time they are quantities going to *vaniſh*. In a third he ſpeaks of the INSTANT of their † *vaniſhing*. In a fourth he ſays, § when the points A and C coaleſce, the rectilinear angle under E A F muſt *vaniſh*. And preſently after, ſpeaking of ſeveral quantities, he takes notice that they ∴ *all vaniſh*, when the point C coincides with A. And in this very *Diſſertation*, he gives us to underſtand that the term vaniſhing, * abſolutely ſignifies no more than going to *vaniſh*: where the participle *vaniſhing* is uſed in his own ſenſe, and the verb *vaniſh* is uſed in mine.

Having now clearly ſettled the meaning of the word vaniſh, both by the authority of Sir *Iſaac Newton*, and that of Mr. *Robins*, I am ready to acknowledge, that when Sir *Iſaac Newton*, inſtead of the expreſſion, *continuo diminuatur, donec tandem evaneſcat*, uſes the term *evaneſcat* ſimply,

* Diſc. p. 50. ‖ Ibid. p. 51.
† Republick of Letters *for* October, p. 257.
§ Ibid. p. 258. ∴ Ibid.
* P. 320.

ply, he intends his reader ſhould thereby underſtand, that the quantity was diminiſhed before it came to *vaniſh*, though the word *vaniſh* does not ſignify diminiſhing. Juſt as if I ſhould ſay, his Majeſty arrived at *Hanover* at ſuch an hour, or Prince *Eugene* died ſuch a day, it would be underſtood that the King was on the way to *Hanover* before he arrived there, and that the Prince lived before he died; and yet arriving does not ſignify the ſame as travelling, nor is dying the ſame as living. But enough of this.

The ſecond error here imputed by Mr. *Robins* to the *Author of the Analyſt*, conſiſts " in ima-
" gining, that by the ultimate ratio of varying
" quantities is meant a ratio, that theſe quanti-
" ties do at ſome time or other exiſt under."

But how a perſon, who ſuppoſes the quantities to vaniſh into nothing, before they arrive at the ultimate ratio, and conſequently makes the ultimate ratio to be a *proportion between nothings*, can be ſaid to imagine, that the *quantities exiſt under that ratio*, is hard to conceive.

And it will be equally difficult to conceive, that a perſon, who makes the varying quantities to vaniſh at the inſtant that they arrive at the ultimate ratio, can be charged with imagining that the *quantities exiſt under that ratio*.

XXVIII. In page 320 we are told, *Philalethes* had no neceſſity, for avoiding the conſequence " charged upon this doctrine, to have recourſe
" to that definition of a *naſcent increment*, which
" follows. *A naſcent increment is an increment juſt*
" *beginning to exiſt from nothing, or juſt beginning*
" *to be generated, but not yet arrived at any aſſign-*
" *able magnitude, how ſmall ſoever.*"

This is very true; and accordingly *Philalethes* did not give this definition *for avoiding* any *conſequence charged upon this doctrine*, but in order to follow Sir *Iſaac Newton*. * *Cave intellexeris particulas finitas. Particulæ finitæ non ſunt momenta, ſed quantitates ipſæ ex momentis genitæ. Intelligenda ſunt principia* JAM JAM NASCENTIA *finitarum magnitudinum.*

XXIX. Mr. *Robins* may ſee by this, that *Philalethes* can read *latin*, and therefore may have *learn'd this doctrine ſolely from Sir* Iſaac Newton's *works*, as well as Mr. *Robins*, may as well be ſuppoſed to have taken the words *naſcent* and *evaneſcent* from Sir *Iſaac Newton* himſelf, as from *thoſe who have writ upon this ſubject in our language*. But if Mr. *Robins* continues to *diſlike* thoſe words, he is welcome, wherever he meets with them, to put in their room the *latin* ones *naſcentes* and *evanaſcentes*. I know of no difference but in the ſound.

XXX. In the next page, Mr. *Robins* ſeems to pleaſe himſelf with having once caught *Philalethes* in a contradiction. It were pity, methinks, to deny him this ſmall gratification, being, perhaps, the only one he is like to meet with in the whole controverſy; and yet juſtice requires I ſhould obſerve, that this ſeeming contradiction ariſes only from ſuppoſing *Philalethes* to *ſuppoſe* what he never had a thought of ſuppoſing, viz. *evaneſcent quantities ſubſiſting at each point of the curve, which can be the ſubjects of the proportion between the ordinate, tangent, and ſubtangent*.

What

* Lemm. 2. Lib. II. Princip.

† Republick of Letters *for* April, p. 321.

What *Philalethes* conſtantly ſuppoſes is, not that thoſe quantities ſubſiſt under this proportion; but the very contrary; that, at one and the ſame inſtant of time, they arrive at this proportion and vaniſh, they vaniſh and arrive at this proportion, as is manifeſt from twenty places of his writings.

XXXI. Mr. *Robin*'s [a] *now proceeds to conſider, what Sir* Iſaac Newton *has called the momenta of quantities.* But, before I enter upon this conſideration, I cannot but take ſome notice of the manner, in which he has been pleaſed to treat Sir *Iſaac Newton*, under pretence of defending him.

Sir *Iſaac*, in ſeveral places of his works, has ſtrongly declared his diſlike of indiviſibles, as ſtrongly as Mr. *Robins* himſelf has done it, and has given his reaſons for that diſlike. And yet this Gentleman, though he occaſionally quotes the very paſſages, where that [b] diſlike, and the [c] reaſons of it, are expreſs'd by Sir *Iſaac Newton*; and likewiſe ſome other paſſages, in which Sir *Iſaac* [d] cautions his reader, that the expreſſions he uſes either for brevity, or the more eaſy conveying his notions, are not to be underſtood in the ſenſe of indiviſibles; notwithſtanding all this, I ſay, Mr. *Robins* is perpetually [e] accuſing him of making uſe not only of the *language* or what *reſembles the language*, but even of the *ſenſe*, the *principles*, the *Method* of indiviſibles.

I muſt however obſerve, in juſtice to that Gentleman, that he did not always treat Sir *Iſaac Newton*

[a] P. 323. [b] Repub. of Let. for *Oct.* p. 265. and Repub. of Let. for *Apr.* p. 292, 295, 311.

[c] Repub. of Let. for *Oct.* p. 265. [d] Repub. of Let. for *Oct.* p. 258, 266. and Repub. of Let. *Apr.* p. 317.

[e] Repub. of Let. for *Oct.* p. 258, 263, 264, 265, 266. and Repub. of Let. *Apr.* p. 311, 312, 324, 327.

Newton in this manner. It ſeems to be with unwillingneſs and reluctance, great neceſſity enforcing him thereto, that he is brought to uſe him ſo unhandſomely.

In the Introduction to his *Diſcourſe* upon Fluxions, having taken notice, that the method of indiviſibles was *obſcure and indiſtinct*, and *obnoxious to error*, he preſently after tells us, " Sir " *Iſaac Newton* to avoid the imperfection, with " which this method of indiviſibles was juſtly " charg'd, inſtituted an Analyſis upon OTHER " principles;" " has taught an Analyſis free " from all obſcurity and indiſtinctneſs;" has " moreover invented a ſynthetick form of rea- " ſoning, which is much more conciſe than the " method of demonſtrating uſed in theſe caſes by " the ancients, yet is equally diſtinct and con- " cluſive;" " has very diſtinctly explained both " theſe ſubjects, his method of fluxions, and his " doctrine of prime and ultimate ratio's, the firſt " in his treatiſe on the quadrature of curves, and " the other in his Mathematical principles of " natural philoſophy; which methods have all " the accuracy of the STRICTEST mathema- " tical demonſtration.

I might produce ſeveral other compliments of the like nature, beſtowed by Mr. *Robins* upon Sir *Iſaac Newton*, in the body of that *Diſcourſe*; nor is there any reflection intermixed, to leſſen the merit of his inventions, except the following; which, if it be written with that deſign, is however expreſſed in a very cautious and tender manner. " In this (the explanation of the term " *momentum*) I ſhall be the more particular, " becauſe Sir *Iſaac Newton*'s definition of mo- " menta, that they are the momentaneous in- " crements or decrements of varying quantities,

" may

"may POSSIBLY be thought obſcure." All this while no mention is made of Sir *Iſaac Newton*'s uſing indiviſibles, or ſo much as any expreſſion *analogous to the phraſeology* of indiviſibles.

But the *Minute Mathematician* happening to be publiſhed a little before Mr. *Robins*'s *Diſcourſe*, and ſome of the beſt judges being of opinion, that *Philalethes* had rightly underſtood Sir *Iſaac Newton*, where Mr. *Robins* had grievouſly miſtaken him, this made it neceſſary to give an account, or rather to publiſh a vindication of the *Diſcourſe* upon fluxions, in the *Republick of Letters* for *October* laſt. In which Mr. *Robins* endeavoured to prove, with what ſucceſs the learned may eaſily judge, that in ſome paſſages he had rightly interpreted Sir *Iſaac Newton*, and that *Philalethes* had miſtaken him. But in ſome other paſſages Mr. *Robins* ſeems to have found himſelf ſo manifeſtly in the wrong, as utterly to deſpair of reconciling his interpretation to what Sir *Iſaac Newton* has delivered. This obliged him, rather than own himſelf at all in an error, to depart from that candid and ingenuous behaviour he had hitherto uſed, with reſpect to our common Maſter. Whenever *Philalethes* produced a deciſive paſſage from Sir *Iſaac Newton*, and urged it home, that there was no poſſibility of evading it; he weakly thought the buſineſs was done, that Mr. *Robins* would give up the point, ſilently at leaſt, if not by an open and ingenuous acknowledgement of his miſtake. But he knew little of his antagoniſt. Indiviſibles was the word; and Sir *Iſaac Newton* himſelf muſt be condemned, rather than *Philalethes* ſhould be acquitted. Let him produce the ſtrongeſt and cleareſt paſſages from Sir *Iſaac*'s Writings, it was only ſaying they were written

written *in the ſenſe of indiviſibles*, and the affair was over.

At firſt, indeed, this charge againſt Sir *Iſaac Newton* was not extended ſo far by a great deal, as it has ſince been carried. Obſerve how modeſtly this point is touched upon in the Month of *October*. " As [f] he thought fit (in compliance " with the bad taſte which then prevail'd) to " continue the uſe of ſome looſe and indiſtinct " expreſſions RESEMBLING thoſe of indiviſi- " bles, for which he has himſelf apologized, he " expreſly cautions us againſt miſinterpreting " him in this manner". Can any thing be more tender of Sir *Iſaac Newton*'s reputation? Mr. *Robins* not only mentions Sir *Iſaac*'s caution, which is true and well known; but an apology likewiſe, which it will be hard to find. Nor can we much complain of what is ſaid ſoon after, [g] " It " muſt be acknowledged, that Sir *Iſaac Newton*'s " deſcription (of momenta) is CAPABLE of " an interpretation too much reſembling the lan- " guage of indiviſibles:" For, if his deſcription is only *capable* of ſuch an interpretation, we may underſtand that it is likewiſe *capable* of another interpretation. In the next page this charge is carried a little farther, " Sir *Iſaac Newton* did " ſometimes make uſe of that method of rea- " ſoning." But even here this uſe of indiviſibles is confined to the *Analyſis* of 1666, to *the treatiſe of Quadratures, meaning as he at firſt writ it, from which we have a tranſcript in Dr.* Wallis,[h] *not as he afterwards corrected it in his own edition of that treatiſe*; and to *one propoſition of his principles of natural philoſophy*. The reſt of Sir *Iſaac*'s works are clear of this imputation, particularly his

[f] Repub. of Let. for *Oct.* p. 258.

[g] Ibid. p. 263. [h] Oper. Vol. II. p. 392.

his treatiſe of *Quadratures*, and the firſt ſection of his *Principia*. For Mr. *Robins* is pleaſed to inform us, that [1] "In his treatiſe of *Quadratures*, "he freed his demonſtrations from this defect, "under which they firſt laboured," and ſoon after we are told, "he at length invented his other "method of prime and ultimate ratios, and there-"by entirely got over that difficulty." And Mr. *Robins* concludes with telling us, "When "he (Sir *Iſaac Newton*) purpoſely deſcribes "theſe methods," (that is, in the treatiſe of *Qua-"dratures* and the firſt ſection of the *Principia*,) "and explains their principles, he is not only "perfectly conſiſtent with himſelf, but has alſo "delivered his meaning with ſuch perſpicuity, "that we need not have recourſe to any defe-"rence for his authority to be fully ſatisfied of "the truth of theſe doctrines."

And yet the infallible brand of indiviſibleiſm, the term infinitely little, which is but once uſed in the *Analyſis* of 1666, and but twice in the propoſition condemned, Prop. 10. L. 11. does frequently occur in the firſt ſection of the *Principia*, together with the expreſſions infinitely great, infinitely greater, infinitely leſs; and is to be found in the admired treatiſe of *Quadratures* itſelf.

If it be ſaid, that theſe two parts of Sir *Iſaac*'s works are excepted from the cenſure, on account of the two ſeveral cautions given by him in the *Scholium* to the firſt ſection, *in ſequentibus nolim indiviſibilia, ſed evaneſcentia diviſibilia intelligi*, and *in ſequentibus cave intelligas quantitates magnitudine determinatas, ſed cogita ſemper diminuendas ſine limite*; which cautions may be ſuppoſed to extend not only to the *Principia*, but to the book of *Qua-*

[1] Repub. of Let. for *Oct.* p. 265.

Quadratures, eſpecially the introduction, which probably was written after the *Principia*; I muſt then ask, ſince in each of theſe cautions the words *in ſequentibus* are inſerted, how comes it, that the firſt propoſition of the ſecond book of the *Principia*, is excepted out of Mr. *Robins*'s act of grace? Is not that propoſition, equally with all the others of the *Principia*, comprehended *in ſequentibus?* A candid reader, although theſe cautions had never been given by Sir *Iſaac Newton*, would eaſily have underſtood in what ſenſe the words, *infinite parvorum*, *infinite parva*, were uſed in that propoſition, by the expreſſions, *minuatur in infinitum*, *evaneſcere*, *nihil erit*, which he ſoon after makes uſe of in the examples to that very propoſition.

And in the *Analyſis* of 1666, where the words, *infinite parvis*, are once uſed, it is equally eaſy to ſee what Sir *Iſaac Newton* meant. For in that ſame *Analyſis* we find the expreſſions above quoted, *in infinitum diminui & evaneſcere, ſive eſſe nihil*; *continuo diminuatur donec tandem evaneſcat*; *diminuetur donec evaneſcat*; *continuo decreſcit donec tandem penitus evaneſcat:* All which expreſſions are conformable to his conſtant doctrine, and are peculiarly his own, being never uſed, that I can learn, by any of the Writers upon indiviſibles.

As in the above mentioned vindication of Mr. *Robins*'s *Diſcourſe upon fluxions*, I found ſeveral paſſages in the *Minute Mathematician* obliquely glanced at, I thought proper to clear up and defend thoſe paſſages in the *Republick of Letters* for *November* following. Where I not only take notice of the *accuracy and caution and perſpicuity with which Sir* Iſaac Newton *has delivered thoſe parts of his works, eſpecially, in which the foundation of his doctrine is laid down*; but with an eye to Mr.

Mr. *Robins*'s charge againſt him, of ſometimes admitting indiviſibles, I uſe the following words, "a faulr, which I am bold to ſay, he was NE-"VER guilty of, and the bare ſuſpicion of which "he has endeavoured to prevent, both here, (in "the *Scholium* to the firſt ſection of the *Princi-*"*pia*) and in many other parts of his writings, "with the utmoſt caution."

This had ſo good an effect, that in Mr. *Robins*'s reply, in *December* following, the accuſation againſt Sir *Iſaac Newton* was entirely dropp'd, as well as ſeveral other parts of the controverſy, and to the reſt I replied in *January*, and Mr. *Robins* was ſilent for ſome months.

But in *April* laſt, that Gentleman, whether upon receiving ſome new reinforcements, or from what other cauſe I know not, not only replied to my paper of *January*; but reſumed the conſideration of the points he had waved in *December*, and introduced many new ones from my former pieces; and carried his charge againſt Sir *Iſaac Newton* much farther than he had ever done before. I muſt needs ſay, he was forced ſo to do, unleſs he would acknowledge himſelf in more than one miſtake.

In this piece, not only the charge againſt the [f] *Analyſis* of 1666, and againſt the [g] account of fluxions publiſhed by Dr. *Wallis*, is renewed, and is extended to the [h] account of the *Commercium Epiſtolicum*; but even the [i] *Scholium* at the end of the firſt ſection of the *Principia* is found fault with, as containing *expreſſions analogous to the phraſeology of the doctrine* of indiviſibles, as well as the firſt [k] corollary to the third *Lemma*.

And

[f] Pag. 323, 326, 328, 330. [g] Pag. 324.
[h] Pag. 323, 324. [i] Pag. 311.
[k] Pag. 312.

And whereas, before, the deſcription of moments, in the ſecond *Lemma* of the ſecond book of the *Principia*, was ſaid only to be *capable of an interpretation too much reſembling the language of indiviſibles*; now we are plainly told, Sir *Iſaac Newton*, " [a] did not ſcruple there to deſcribe moments " according to the ſenſe of that doctrine, as he " had done formerly, to be *incrementa vel decre-* " *menta momentanea.*" And in another place we are informed, that Sir *Iſaac*, " [b] ſpeaking of " theſe *momenta*, as *incrementa momentanea*, in the " ſenſe of indiviſibles, ſays, *particulæ finitæ non ſunt momenta.*" Where, I muſt not omit to obſerve, that this paſſage, badly written as it is, in *the unintelligible jargon of indiviſibles*, yet ſerves well enough to be quoted againſt *Philalethes*. It is alſo affirmed, [c] " That Sir *Iſaac Newton* has " made uſe of indiviſibles in the very ſenſe of *Ca-* " *valerius*, and that the doctrine of moments was " originally founded on them", nay, this is ſaid to have been " proved from his own words". We are likewiſe told, [d] that " in the Lectures " he read at *Cambridge* on his admirable diſcove- " ries in Opticks, he uſed indiviſibles in his de- " monſtrations".

To all which methinks the following anſwer were ſufficient, though upon occaſion a more particular reply may be given.

Sir *Iſaac Newton*, in ſeveral parts of his works, particularly in the *Scholium* at the end of the firſt ſection of the *Principia*, and in the introduction to the *Quadratures*, has with great care diſtinguiſhed between his own method and the method of indiviſibles. He was ſo far from making uſe of this latter method, that in all his works,

[a] Republick of Letters *for* April, p. 324.
[b] Ibid. p. 327. [c] Ibid. p. 326. [d] Ibid. p. 329.

works, I may venture to aſſert, there is no paſſage, no expreſſion to be found, that ſhould reaſonably make him ſo much as ſuſpected of it. For, although he may ſometimes uſe expreſſions, that, ſingly taken, are *capable of* ſuch *an interpretation*; yet thoſe expreſſions are capable likewiſe of a very different interpretation. And ſurely, if a writer makes uſe of a term or expreſſion, in itſelf capable of two different ſenſes; yet if one of thoſe ſenſes be agreeable to what goes before and after, and the other be directly contrary to it; if one ſenſe be conformable to the conſtant tenour of his doctrine, and the other manifeſtly contradict it; all the reaſonable and impartial part of the world will agree, that the term or expreſſion ought to be taken in the former ſenſe, and not in the latter: Eſpecially, if the author hath cautioned his reader againſt underſtanding him in the latter ſenſe; more eſpecially ſtill, if ſuch caution hath been often repeated.

By the term *momenta*, Mr. *Robins* * ſeems of opinion, that Sir *Iſaac Newton* intended finite quantities. I maintain, that a moment, in the ſenſe of Sir *Iſaac Newton*, is a momentaneous increment or decrement of a flowing quantity, leſs than any finite quantity whatſoever, and proportional to the velocity of the flowing quantity.

As this diſpute firſt aroſe from the demonſtration of the ſecond *lemma*, *Lib.* 2. *Princip.* methinks it were better for Mr. *Robins* and me, if we aim at truth alone, to confine ourſelves to what is ſaid in that *lemma*, than to amuſe ourſelves and our readers with hunting over Sir *Iſaac*'s other works, and examining every paſſage where the term *momentum* happens to be made uſe of: Eſpecially, as in thoſe paſſages that term is only

* Repub. of Let. *Apr.* p. 325, 327.

tranſiently mentioned, but is here very particularly and diſtinctly explained, and proper cautions given againſt any poſſibility of miſunderſtanding it.

However, ſince Mr. *Robins* chuſes to conſider, how this term was uſed by Sir *Iſaac Newton*, not at the time when he wrote the demonſtration in queſtion, but twenty years before that time; what he underſtood by it in the *Analyſis* of 1666, rather than in the *Principia* publiſhed in 1687; republiſhed in 1713, and again in 1726, the year before his death, without altering this *lemma*; ſo that we may look upon it as containing his laſt and moſt ſettled thoughts upon this ſubject; I am content to follow this Gentleman whither he thinks fit to lead me; but the better to prepare my reader for making a right judgment in this diſpute, I muſt ask leave to make three or four previous obſervations.

The firſt is, that, at the beginning of this *Analyſis*, Sir *Iſaac* has advertiſed his reader, that the method he there delivers, is *rather briefly explained, than accurately demonſtrated:* And juſt before he firſt introduces the term *momentum*, he has this expreſſion. *In iſtis autem quo ego operor modo dicam breviſſime.* And accordingly he takes the latter of the *two ways whereby*, as Mr. *Robins* informs us, *good writers explain the uſe of the terms they introduce: One by expreſly defining them; another, when to avoid that formality, they convey the ſenſe of ſuch terms by their manner of uſing them.*

But the Author of the account of the *Commercium Epiſtolicum*, ſaid by Mr. *Robins* to be Sir *Iſaac Newton* himſelf, acquaints us that, *In the ſecond lemma of the ſecond book of the principles*

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the elements of this calculus are demonſtrated ſynthetically. For which reaſon alone, if there were no other, methinks it were better to content ourſelves with what is ſaid in this *lemma*, than to have recourſe to that *Analyſis*.

Secondly, Mr. *Robins* takes it for granted, that, when Sir *Iſaac Newton* uſes the words *infinitely little*, he always means an indiviſible. But I affirm, that, in Sir *Iſaac*'s works, that expreſſion never ſignifies an indiviſible, except only when he uſes it in condemning indiviſibles. For, although the expreſſions infinitely little and infinitely great are ſometimes uſed by Sir *Iſaac Newton*, as well as by the Writers upon indiviſibles; yet the ſenſe, in which he uſes thoſe expreſſions, is very different from what they underſtand them in.

By an infinitely little quantity, I apprehend them to mean a quantity actually exiſting, fixed, determinate, invariable, indiviſible, leſs than any finite quantity whatſoever. And I agree with Mr. *Robins* that is nonſenſe, it implies a contradiction. I can have no conception of a quantity leſs than any quantity I can conceive. To aſſert this were to ſay, that a quantity may be conceived to be leſs than itſelf.

But Sir *Iſaac Newton* by an infinitely little quantity means a variable, diviſible quantity, that, by a conſtant diminution, is conceived to become leſs than any finite quantity whatſoever, and at laſt to vaniſh into nothing.

And by a quantity infinitely great, I take the Writers upon indiviſibles to mean, a quantity actually exiſting, fixed, determinate, invariable, immutable, greater than any finite quantity whatſoever. Which is likewiſe a contradictory and impoſſible idea. I can have no conception of a

quantity greater than any quantity I can conceive. To ſay this were to aſſert, that a quantity may be conceived to be greater than itſelf.

But Sir *Iſaac Newton*, by a quantity infinitely great, means a variable quantity, that, by a conſtant increaſe, may be conceived to become greater than any finite quantity whatſoever.

Thirdly, I muſt take notice, that Sir *Iſaac* ſometimes ſpeaks of theſe *momenta* as naſcent quantities, and ſometimes as evaneſcent. In which ſoever of theſe lights they are conſidered, it anſwers the ſame end.

But it will be of uſe to obſerve, that when the *momenta* are conſidered as naſcent, they may, by flowing from the inſtant of their origin, during a finite time, arrive at a finite magnitude. During that finite time their proportion conſtantly varies: And it is only at the firſt inſtant of that finite time, that they are in the ſame proportion with the fluxions. The proportion of the fluxions is the proportion with which the momenta, do *naſci*, begin to exiſt.

And when the *momenta* are conſidered as evaneſcent, we may conceive them to have been once of a finite magnitude, and by decreaſing for a finite time, at laſt to vaniſh into nothing. During that finite time their proportion conſtantly varied: And it is only at the laſt inſtant of that finite time, that they arrive at the ſame proportion with the fluxions. That is the proportion with which they vaniſh, or ceaſe to exiſt.

In the former of theſe lights they are conſidered by Sir *Iſaac*, in the *lemma* and demonſtration in queſtion. They are ſpoke of in the ſecond light, in the *Quadratura Curvarum*, and the *Analyſis* of 1666. And in the firſt ſection of the *Principia*, particularly in the *Scholium* at the end

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of that ſection, and likewiſe in the introduction to the *Quadratures*, they are conſidered ſometimes as naſcent, ſometimes as evaneſcent. From not rightly attending to the ſenſe of thoſe paſſages, where the *momenta* are conſidered as having once been finite quantities, and being now by a conſtant diminution arrived at the point of evaneſcence, I apprehend Mr. *Robins* to have been led into the groſs miſtake, of taking the *momenta evaneſcentia* for finite quantities.

Fourthly, I muſt take leave to put Mr. *Robins* in mind of what he himſelf has quoted from the *Phil. Tranſ.* as the words of Sir *Iſaac Newton*. "In his calculus there is but one infinitely lit-" tle quantity repreſented by a ſymbol, the ſym-" bol *o*.

" Mr. *Newton* uſed the letter *o* in his *Analyſis*, " and in his book of *Quadratures*, and in his " *Principia Philoſophiæ*, and ſtill uſes it in the " very ſame ſenſe as at firſt."

From which it follows, that we need only conſult the *Principia*, or the book of *Quadratures*, to ſee in what ſenſe the letter *o* is there uſed: By that we ſhall know the ſenſe it is uſed in, in the *Analyſis*. In which ſoever of theſe three treatiſes, the *momenta* are moſt fully and clearly deſcribed, it is to that undoubtedly we ought to have recourſe, in order to ſee in what ſenſe they are uſed in all the three. We muſt therefore have recourſe to the *Principia*, and particularly to the ſecond *lemma*, *lib.* 2. where only there is any deſcription of them given. In the other of theſe treatiſes they are tranſiently mentioned, but no deſcription is given, that I know of.

This being premiſed, I come to make out my deſcription of a moment, from what Sir *Iſaac New-*

ton ſays in this *lemma*. That deſcription conſiſts of three parts.

1. A moment is a momentaneous increment or decrement of a flowing quantity; *Has quantitates ut indeterminatas & inſtabiles, & quaſi motu fluxuve perpetuo creſcentes vel decreſcentes hic conſidero; & earum incrementa vel decrementa momentanea ſub nomine momentorum intelligo;*

2. Is leſs than any finite quantity; *Cave tamen intellexeris particulas finitas. Particulæ finitæ non ſunt momenta, ſed quantitates ipſæ ex momentis genitæ. Intelligenda ſunt principia jamjam naſcentia finitarum magnitudinum.*

3. And is proportional to the velocity of the flowing quantity. *Neq; enim ſpectatur in hoc lemmate magnitudo momentorum, ſed prima naſcentium proportio. Eodem recidit ſi loco momentorum uſurpentur vel velocitates incrementorum, vel finitæ quævis quantitates velocitatibus hiſce proportionales.* To make this perfectly clear, we may obſerve, that Sir *Iſaac*'s intention is only to take the firſt proportion of the naſcent moments: And when he tells us, it does *eodem recidere*, his meaning is, that the very ſame proportion will be obtained, whether we take, for the terms of our proportion, the moments, or the velocities, or any finite quantities whatſoever proportional to thoſe velocities, *i. e.* that the moments are proportional to the velocities.

It will perhaps be ſaid, that, as the quantities here conſidered by Sir *Iſaac*, are indeterminate and inſtable, and increaſing by a perpetual flux or motion; ſo likewiſe their momentaneous increments, or moments, are to be conſider'd as indeterminate and inſtable, and perpetually flowing or increaſing; and conſequently will become finite quantities, as Mr. *Robins* ſuppoſes them to be. But

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to this I anſwer, that, altho' the moments will by flowing become finite; yet, as we have taken notice in our third obſervation, when they are become finite, they will not have the ſame proportion as the velocities, the proportion of the velocities being that of the naſcent moments only, being only that with which the moments begin to exiſt. If you take their proportion at any other inſtant, than that of their origin, it will not *eodem recidere*, as if you take the proportion of the velocities.

So likewiſe, if the moments, after becoming finite quantities, be ſuppoſed again to decreaſe, and by a conſtant diminution at laſt to vaniſh into nothing; their variable proportion, while they are finite quantities, will be different from that of the velocities; and they will arrive at the proportion of the velocities, at the inſtant of their vaniſhing only, or the evaneſcent moments only will be proportional to the velocities.

Perfectly agreeable to this explication is the paſſage above quoted * from the account of the *Commercium Epiſtolicum:* But as this paſſage has been already ſufficiently explained, and has been applied to vaniſhing quantities in general, there is no need of being particular in the application of it to evaneſcent moments.

From what has been ſaid upon this ſubject, it ſeems very plain, that the *momenta* deſcribed in this *lemma* are not finite quantities, nor yet indiviſibles: And as theſe *momenta* were always uſed by Sir *Iſaac Newton* in the ſame ſenſe as at firſt, it follows that the *momenta* in the book of *Quadratures* and in the *Analyſis* of 1666, are the very ſame as thoſe of this *lemma*. Therefore from this alone, although we had given no other argu-

* Repub. of Letters for *July*, p. 54.

ment, we may ſafely conclude againſt Mr. *Robins* that the *momenta* of that *Analyſis*, though called infinitely little, are not indiviſibles. It only remains to conſider one or two paſſages, that Mr. *Robins* has produced in favour of his opinion.

XXXII. The firſt is taken from the account of the *Commercium Epiſtolicum*, and is thus quoted by Mr. *Robins*. * "He (Sir *Iſaac Newton*) ſays "that *he there called the moment of a line a point* "*in the ſenſe of* Cavalerius, *and the moment of an* "*area a line in the ſame ſenſe.*"

Now the paſſage itſelf runs thus, † "The "moment of a line he called a point in the ſenſe "of *Cavallerius*, THO' it be not a geometrical "point, but a line infinitely ſhort, and the mo- "ment of an area or ſuperficies he called a "line in the ſenſe of *Cavallerius*, THO' it be "not a geometrical line, but a ſuperficies in- "finitely narrow."

If my reader ſhould be curious to enquire, for what end or purpoſe Mr. *Robins* choſe rather to truncate this paſſage, in the manner above ſet down, than to give it entire; I can eaſily inform him. He was willing it ſhould be thought, that, in this *Analyſis*, Sir *Iſaac Newton* meant the very ſame thing by the moment of a line, as *Cavallerius* meant by a point; and that Sir *Iſaac*, by the moment of a ſurface, meant the very ſame thing, as *Cavallerius* meant by a line. For he does not ſcruple to tell us, a page or two after, ‖ "That Sir *Iſaac Newton* "has made uſe of indiviſibles, in the very ſenſe "of *Cavallerius*, and that the doctrine of mo- "ments was originally founded on them, we "have already proved from his own words."

Where

* Rep. of Lett. for *Apr.* p. 323.
† Phil. Tranſ. N°. 342. p. 178. ‖ P. 326.

Where he intends this very paſſage, taking it for granted, that the Account of the *Commercium Epiſtolicum* was written by Sir *Iſaac Newton.*

But, from the paſſage taken entire, it plainly appears, that, what Sir *Iſaac* meant, by the moment of a line, was not the very ſame as *Cavallerius* meant by a point, but was *a line infinitely ſhort*; and that what Sir *Iſaac* meant by the moment of a ſurface, was not the very ſame as *Cavallerius* meant by a line, but was a ſuperficies infinitely narrow: And conſequently Sir *Iſaac Newton has* not *made uſe of indiviſibles in the very ſenſe of* Cavallerius.

All that is intended by theſe words, *in the ſenſe of* Cavallerius, is no more than this: What Mr. *Newton* calls the moment of a line, would by *Cavallerius*, or any follower of *Cavallerius*, who wrote after his manner, and ſpoke his *ſenſe*, be called a point; though it is not a point, but a line. And what Mr. *Newton* calls the moment of a ſurface, would alſo by *Cavallerius*, or his followers, be called a line; though it is not a line, but a ſurface.

Juſt after the ſame manner, it is ſaid, * *xo* and *dx*, ſignify the ſame moment; and in other places moments or differences are ſpoken of, as equivalent terms, ſignifying only that what Sir *Iſaac Newton* calls moments in his method, anſwer to what are called differences in the method of Monſieur *Leibnitz*. So alſo the method of fluxions, and the differential method of Monſieur *Leibnitz*, are ſaid to be one and the ſame method, in the report made to the *Royal Society* by the Committee appointed to examine the old letters and papers, of which an account is given in the *Com-*

* Phil. Tranſ. N°. 342. p. 204.

Commerc. Epiſtol. tho', as Sir *Iſaac Newton* obſerves in the *Scholium* at the end of *Lemm.* 2. *Libr.* 11. *Princip.* Mr. *Leibnitz*'s method differs from that of fluxions, *in verborum & notarum formulis*, and alſo in the *idea generationis quantitatum*, Mr. *Leibnitz* conceiving quantities to be compoſed of differences added together, and Sir *Iſaac Newton* conceiving them to grow and increaſe by a continual flux.

Mr. *Robins*, after the above-mentioned quotation, tells us, " The paſſage in the book, to " which this relates, is as follows. *Nec vereor* " *loqui de unitate in punctis, ſive lineis infinite par-* " *vis, ſiquidem proportiones ibi jam contemplantur* " *Geometræ, dum utuntur methodis indiviſibilium.*"

This, I think, is not true. For the paſſage, to which the quotation relates, is manifeſtly the following. " Sed notandum eſt quod unitas iſta " quæ pro momento ponitur eſt ſuperficies cum " de ſolinis, & linea cum de ſuperficiebus, & " punctum cum de lineis (ut in hoc exemplo) " agitur." In which paſſage the moment *o* is underſtood, though not expreſs'd, as is declared in the account of the *Comm. Epiſtolicum*.

XXXIII. The next paſſage alledged by Mr. *Robins* is this, " becauſe we have no ideas of in- " finitely little quantities, he (Sir *Iſaac Newton*) " introduced fluxions into his method, that it " might proceed by finite quantities as much as " poſſible."

I do not well apprehend what Mr. *Robins* would infer from this paſſage. If he means that Sir *Iſaac Newton* introduced fluxions on purpoſe to avoid indiviſibles, I agree with him. But then I muſt obſerve, that Sir *Iſaac* had already invented the method of fluxions, when he wrote the

the *Analyſis*, as Mr. *Robins* himſelf * acknowledges. Conſequently, he had then found a way of avoiding indiviſibles; and when he had invented fluxions on purpoſe to avoid indiviſibles, it cannot reaſonably be ſuppoſed he would ſtill uſe indiviſibles.

So far was he from this, that he was for *proceeding by finite quantities as much as poſſible*. And accordingly we find him uſing no other than finite quantities, except that he ſuppoſes theſe finite quantities to diminiſh *ad infinitum* and vaniſh, in order to find their laſt proportions.

XXXIV. But we are told, "he had not at "the firſt diſcover'd his doctrine of prime and "ultimate ratios, which entirely rejects indivi-"ſibles."

I am glad with all my heart to hear, that *the doctrine of prime and ultimate ratios entirely rejects indiviſibles*. Surely, it will not now be pretended, that the ſecond *lemma* of the ſecond book of the *Principia*, or any part of it, is written *in the ſenſe of indiviſibles*; much leſs that any part of the firſt ſection of the *Principia*, is written in ſuch a ſenſe. As the only deſign of that *lemma* is to determine the firſt proportion of the naſcent moments, and as the ſole buſineſs of the firſt ſection of the *Principia* is to deliver and explain the doctrine of firſt and laſt ratio's in general, indiviſibles muſt in both theſe be *entirely rejected*.

But is it true, that, *at firſt*, when the *Analyſis* of 1666 was written, Sir *Iſaac Newton had not diſcovered his doctrine of prime and ultimate ratios?* I believe not.

Mr. *Robins* tells us a few pages farther, "We "are ſure he had a part of it in 1669, on ac-"count

* Rep. of Let. for Apr. p. 328.

" count of a demonſtration added at the end of " his *Analyſis.*" This is ſomething. But I will tell Mr. *Robins* how he may be *ſure*, very ſure, that Sir *Iſaac* had the whole of this method, at the time when he wrote that *Analyſis*.

That Mr. *Robins* is poſſeſſed of the whole of this doctrine, no doubt can be made, after he has ſo *eaſily and elegantly* explained it in his *Diſcourſe* upon that ſubject, *and that by a method not yielding in brevity to the artleſs inconcluſive proceſs by indiviſibles*. I would therefore be glad to know of him, what courſe he would take, to determine the prime and ultimate ratio of the *momenta* mentioned in the *Analyſis* in queſtion.

I ſuppoſe he will tell me, his anſwer is already written. He will take the ſame courſe, as Sir *Iſaac* has taken * *in his treatiſe of Quadratures*, where *he* has *freed his demonſtrations from this defect*, (the uſe of indiviſibles) *under which they firſt laboured*. He will ſuppoſe his moments to be at firſt finite, and having finiſhed his calculation, he will then ſuppoſe the moments to be diminiſhed *ad infinitum* and vaniſh into nothing, and will thereby obtain the laſt proportion of thoſe moments.

But is not this the very method taken in the *Analyſis* of 1666? Does not the Writer of the account of the *Commercium Epiſtolicum*, that is, Sir *Iſaac Newton* himſelf, as Mr. *Robins* acquaints us, does he not, I ſay, diſtinctly inform us, in the paſſage quoted from that account by Mr. *Robins* in the next † page, that this very method was taken in that *Analyſis?* Does not he alſo ſay in the ſame account, p. 182? " This was Mr. *Newton*'s " way of working in thoſe days, when he wrote " this

* Rep. of Lett. for *October*, p. 265.
† Rep. of Lett. for *April*, p. 324.

" this compendium of his *Analyſis?* And the " SAME way of working he uſed in his book of " Quadratures, and STILL uſes to this day." Do we not accordingly ſee, that, in the demonſtration of the firſt rule of that *Analyſis*, Sir *Iſaac* takes exactly the ſame method, and almoſt in the ſame words, as in the demonſtration of the firſt Propoſition of the *Quadratures?* If it be ſaid, there is no mention of prime and ultimate ratios in the firſt of theſe demonſtrations; we anſwer, neither are they mentioned in the other. But the method is the ſame. Sir *Iſaac* could find the firſt and laſt proportions of variable quantities, without deafning our ears with the ſound of prime and ultimate ratio's a thouſand times repeated. This is eaſily ſeen not only in this *Analyſis*, but in the tenth propoſition of the ſecond book of the *Principia*, by thoſe who attend more to ſenſe than ſound, more to the meaning than the form of expreſſion.

XXXV. Had Mr. *Robins* done thus, he would not have fallen into the miſtake that follows, of thinking Sir *Iſaac Newton* uſed the word moment in * two ſenſes, becauſe, in demonſtrating, he at firſt ſuppoſes a moment to be finite, and afterwards to decreaſe *in infinitum* and vaniſh; and in inveſtigation, for making diſpatch, he at once ſuppoſes his moment to be infinitely little. For this infinitely little moment is no other, than the former moment conſidered as diminiſhed *in infinitum* and vaniſhing, as has been ſhown above more than once.

XXXVI. Mr. *Robins* informs us, that Sir *Iſaac Newton* † " thought fit for the ſake of " brevity to introduce this term moment in the ſecond book of his *Principia Philoſophiæ.*" The place

* Repub. of Let. for Apr. p. 324. † Ibid.

place here meant is the ſecond *lemma* of the ſecond book; where the doctrine of fluxions is likewiſe introduced, but for a different reaſon, according to Mr. *Robins*. * "It ſeems, ſays this "Gentleman, as if he took notice of them chiefly, that a cypher, he thought fit to explain relating to them, might be underſtood."

I apprehend, Sir *Iſaac* introduced the mention of fluxions here, in order to explain and demonſtrate the foundation of that method, and that he introduced moments in order to ſerve for that foundation, his method of fluxions being built upon the proportions of thoſe moments.

But let it be ſo, that moments were introduced *for the ſake of brevity*, and fluxions for the ſake of *a cypher*, what are we next to believe?

We are told, † Sir *Iſaac Newton* "did not "ſcruple there to deſcribe moments according to "the ſenſe of that doctrine" (of indiviſibles.) But, "he ſhews how to correct the idea ariſing "from this deſcription of moments."

Let us therefore for once, notwithſtanding all that has been ſaid to the contrary, admit this to be true, that Sir *Iſaac*'s *momenta*, his *incrementa vel decrementa momentanea*, are *deſcribed according to the ſenſe of the doctrine of indiviſibles*. If the words, *incrementa momentanea* give us the idea of indiviſibles, how is this idea to be *corrected?*

Mr. *Robins* gives the corrective to this idea in two parts.

The firſt part of the corrective is, "You muſt "never conſider their magnitudes, but their ultimate ratio." Now, how this corrective is to operate, in order to amend the idea already conceived of indiviſible, or infinitely little quantities, I can by no means underſtand. If I muſt never conſi-

* Rep. of Let. for *April*, p. 330. † Ibid. p. 324.

conſider the magnitude of the moments, how is it poſſible for me to know, that their magnitude is either finite, or infinitely little? This firſt part of the corrective ſeems to be good for nothing; I ſhall never get rid of my idea of infinitely little quantities by theſe means.

The ſecond doſe of Mr. *Robins*'s corrective looks much more promiſing. It runs thus. "It "would come to the ſame thing, if inſtead of "theſe moments you uſed the velocities of in- "creaſe or decreaſe of quantities, which he (Sir "*Iſaac Newton*) is wont to call fluxions, or if "you uſed any OTHER finite quantities propor- "tional to theſe fluxions."

The efficacy of this remedy depends entirely upon a right uſe of the word, *other*. If this word is to be underſtood in contradiſtinction to the word velocities, that we may uſe velocities, or fluxions, (which are finite quantities) or any *other* finite quantities proportional to theſe fluxions; the corrective is of no manner of uſe, it will never rectify the idea I have entertain'd of moments being indiviſibles. But it happily falls out, that this is by no means the natural purport of the words, any more than it can at all ſuit Mr. *Robins*'s deſign.

Muſt we not therefore interpret the paſſage thus, that, inſtead of theſe moments we may uſe any *other* finite quantities proportional to the fluxions? This indeed is clear and deciſive, and manifeſtly implies that moments are finite quantities. Otherwiſe, it were abſurd to ſay, inſtead of moments, any *other* finite quantities may be uſed. Conſequently, ſince moments are thus ſhewn to be finite quantities, they cannot be infinitely little quantities, the idea of indiviſibles is wholly removed, aboliſhed, eradicated and extirpated.

But,

But, alas! it unfortunately happens, that this word *other*, this efficacious, this important word is no where to be found in Sir *Iſaac Newton*'s Preſcription. Though he has the honour of it given him, yet, it is certain, it never entered the good old gentleman's thoughts, at leaſt it never dropp'd from his pen. His words are, not *finitæ quævis* ALIÆ *quantitates*, but *finitæ quævis quantitates*. And conſequently the moments muſt ſtill continue infinitely little, we may be *perpetually obnoxious to the abſurdities of the doctrine of indiviſibles*, notwithſtanding any aſſiſtance from this corrective.

The reader perhaps will be curious to know, how this word came to be ſlipt into the quotation from Sir *Iſaac Newton*. And poſſibly, if he is not throughly acquainted with Mr. *Robins*'s integrity and candor, he may be apt to ſuſpect that Gentleman of ſome foul play. If ſo, I muſt undeceive him; though, I am ſenſible, it will be hard to bring him to believe, that the inſertion of this word *other*, which ſo totally alters Sir *Iſaac*'s ſenſe to Mr. *Robins*'s purpoſe, is owing only to the zeal of an officious Compoſitor for the preſs. And yet I cannot but acquaint him, that, no ſooner was notice given by a friend of this extraordinary proceeding, which was like to be interpreted very much to Mr. *Robins*'s diſadvantage, than care was taken to rectify it, among the *Errata* of the preſs, in the *Republick of Letters* for the following month; where any one, who doubts of my veracity, may with his own eyes ſee this correction made, p. 325. lin. 8. *dele* other.

And when I have appriſed that Gentleman, that there is a paragraph towards the bottom of the ſame page, which calls the above-mentioned permiſſion of Sir *Iſaac Newton* by the name of a *caution*, and pretends that by *a* and *b* he meant finite

nite quantities, that is, as I underſtand, finite quantities only; which paragraph manifeſtly proceeds from the ſame hand, as had before inſerted the word *other*, and is grounded ſolely upon that inſertion; no doubt ought to be made, but that he will take care, for his own reputation, to place this whole paragraph likewiſe among the *errata*.

If any one ſhould object how incredible it is, that a whole paragraph, which cannot poſſibly paſs for an *Erratum*, and ſuch a paragraph too, ſhould be put in by a Compoſitor; I muſt deſire him to obſerve, that this is no common Compoſitor: He muſt be a conſiderable Mathematician, as well as a Compoſitor; elſe he could never have been able to put in ſo pertinent a word, and ſo cleverly ſuited to his Author's purpoſe, as the word OTHER. And I muſt likewiſe obſerve, that his calling that a caution, which any man of learning would rather call a permiſſion, or liberty given us by Sir *Iſaac Newton*, is much more like a Compoſitor, than a perſon ſo *exact in the choice of his expreſſions* as Mr. *Robins*. For in that paſſage Sir *Iſaac* does not *caution* us to uſe fluxions, or finite quantities proportional to them, inſtead of moments; he does not tell us we muſt uſe them; but gives us to underſtand we may uſe them; tells us it will come to the ſame thing, if we do uſe them.

I wiſh this ſame Compoſitor may not have impoſed upon us in other places, particularly in the words *literal tranſlation*, *coincidence* and *coinciding* in *December*, and *literally import* in this preſent *Diſſertation*, p. 302. as likewiſe in the word *always*, p. 321. but eſpecially in the words, *or neceſſarily imply*, p. 306. *'cum multis aliis.*

XXXVII. Mr. *Robins* comes next to object to my demonſtration, as he is pleaſed to call it, of

the method for finding the *momentum* of a rectangle.

He ſays, I "have endeavoured to prove, that "the moment of the rectangle is an arithmetick "mean proportional between the contemporaneous increment and decrement of the ſame "rectangle. But it has been ſhewn, that this is "only true, when the ſides augment in the ſame "conſtant proportion."

I do not call to mind, that I have any where ſpoke of *the increment and decrement of the ſame rectangle*, as *contemporaneous*. If I have done ſo, I was certainly not very *exact in the choice of* my *expreſſions*. The increment of one rectangle may be contemporaneous with the increment of another rectangle: But the increment of a rectangle cannot poſſibly be contemporaneous with the decrement of the ſame rectangle. However, let that paſs.

I imagine, Mr. *Robins*'s meaning is, that ſuppoſing the moment, increment and decrement to be ſeverally generated in equal particles of time, I have endeavoured to prove, that, in this caſe, *the moment is an arithmetick mean proportional between the increment and decrement*.

But, ſays he, *it has been ſhewn, that this is only true, when the ſides augment in the ſame conſtant proportion.*

Let it be ſo; at leaſt, in the caſe of finite moments. But, in that *ſuppoſed demonſtration*, I never conſidered any caſe, where *the ſides* do not *augment in the ſame conſtant proportion*. I there conſidered only that one caſe, in which the ſides augment uniformly.

I was not then writing a *Diſcourſe upon Fluxions*; but an anſwer to the *Author of the Analyſt*. And that Author did not then appear to me, nor does he yet appear, after his reply to me, and his

his ſeveral anſwers to Mr. *Walton*, ever to have conſidered any other caſe, in his two objections to Sir *Iſaac Newton*, than that only, in which the ſides are uniformly augmented.

I told him therefore, the moment of the rectangle *AB* was different from the increment of the rectangle *AB*. And from his own poſitions, or ſuch as by parity of reaſon he was obliged to acknowledge equally with his own, I * ſhewed him, that the increment of the rectangle *AB* added to the decrement of the ſame rectangle, made twice the moment of that rectangle; and conſequently that the moment of the rectangle *AB* was $aB + bA$, the ſame as Sir *Iſaac Newton* had determined. But throughout that argument there is no ſuch word as demonſtrate, or demonſtration to be found; and the whole tenour of the paſſage ſhews it to be addreſſed to the *Author of the Analyſt*, and to him only.

In the *Minute Mathematician* publiſhed a year afterwards, I have indeed called it a demonſtration, nay a demonſtration as ſtrong as any in *Euclid*: but pray, good reader, mind the words. † "It is;" ſaid I, addreſſing myſelf to the *Author of the Analyſt*, "a demonſtration AGAINST "YOU as ſtrong as any in *Euclid*, that the mo- "ment of the rectangle *AB* is a middle arithme- "tical proportional between the increment and "decrement of the ſame rectangle *AB*."

By the words, *againſt you*, it is eaſy to ſee, this was not intended for a demonſtration, ſtrictly ſpeaking; but for an *argumentum ad hominem*, and nothing elſe. And, if I am not greatly miſinformed, Mr. *Robins* himſelf was once of this opinion; inſomuch that, after he had inſerted this very objection into his paper of *October*

* Geometry no Friend to Infidelity, p. 46. † P. 51.

laſt, upon ſhewing it to a friend of his, who is likewiſe a lover of truth, which friend told him of theſe words and that they manifeſtly imported a meer *argumentum ad hominem*, he ſtruck the whole objection out of his copy. How comes it now to be put in? Does he think the words, *againſt you*, were ſlipt in by the compoſitor? He will not find them among the *Errata*.

This proceeding of Mr. *Robins* ſeems to be the more extraordinary, becauſe a perſon of his penetration might eaſily ſee, that, although in this *ſuppoſed demonſtration* I had conſidered only one caſe, yet I had the other caſes in my eye, and particularly was prepared to deduce the ſecond caſe from the firſt by the help of the method of laſt proportions, whenever I ſhould be called upon ſo to do, from what I had written in page 50, 53 of the ſame *Letter*; but eſpecially after the declaration I had made in the *Minute Mathematician* p. 81, that *this was to guard againſt future objections*. At leaſt, as the defect of this demonſtration, if there were any in it, has ſince been ſupplied by a fuller and clearer demonſtration publiſhed in *November* laſt, the former might well have been ſpared by Mr. *Robins*.

XXXVIII. In this laſt demonſtration I have made uſe of finite moments, and hope therefore it is entirely to Mr. *Robins*'s mind. I ſee he has carefully peruſed it, by his nibbling at two or three paſſages, which, though no way material to the truth or clearneſs of that demonſtration, I ſhall nevertheleſs, out of regard to the Objector, particularly conſider.

Firſt, Mr. *Robins* charges me with ſaying, * *that Sir* Iſaac Newton *conſiders theſe moments as the differences of* Leibnitz.

I have

* Republick of Letters *for* April, p. 326.

I have ſaid no ſuch thing. What I have ſaid is, † "There is likewiſe a third acceptation, in "which we may take the quantities repreſented "by *a*, *b*, *c*. For theſe LETTERS may be "underſtood to repreſent the differences of "Monſieur *Leibnitz*." I had before taken notice, that thoſe letters were by Sir *Iſaac Newton put to ſignify either the moments, or the velocities, of the flowing quantities A, B, C.*

But if I had really done what Mr. *Robins* here imputes to me; all that could be meant by it is, that theſe *momenta* are conſidered as analogous to the differences of Monſieur *Leibnitz*; not that they are exactly the ſame with thoſe differences. And here, were it neceſſary, I could ſhelter myſelf under the authority of a writer, who makes no little figure in the *Republick of Letters*, who has likewiſe given the name of *momenta* to Monſieur *Leibnitz*'s differences. "In * that me-"thod," (of indiviſibles) ſays he, "theſe *mo-"menta* are conſidered abſolutely as parts, where-"of their reſpective magnitudes are actually com-"poſed." Where it is obſervable, that, in three ſentences, immediately following one another, the very ſame words, *theſe momenta*, are made uſe of. And yet in the firſt ſentence, by *theſe momenta*, he means the *momenta* of Sir *Iſaac Newton*; in the ſecond, by theſe *momenta*, he means the differences of Mr. *Leibnitz*; and in the third, by *theſe momenta*, he again means the *momenta* of Sir *Iſaac Newton*.

How other perſons may be affected with ſuch paſſages, I know not. For my ſelf, I can never read them without joining in the lamentations of Mr. *Robins*, while, like another *Heraclitus*, or rather like a ſecond *Jeremy*, he weeps over the *Geometrical Writers* and Sir *Iſaac Newton*.

† Republick of Letters for *November*, p. 386.
* Diſcourſe upon Fluxions, p. 77.

——————— *Quis talia fando*
Leibnitius, Greniuſve, *aut duræ Præſul* Iernes
Temperet a lachrymis! Manibus date lilia plenis;
Tu Newtonus *eras.*

How mournfully does he bewail their *negligence in regard to their ſtyle and diction!*

Qualis populeâ moerens Philomela ſub umbrâ
Amiſſos queritur foetus, quos durus arator
Obſervans nido implumes detraxit, at illa
Flet noctem, ramoque ſedens miſerabile carmen
Integrat, & moeſtis latè loca queſtibus implet.
Mirandum eſt unde ille oculis ſuffecerit humor!

XXXIX. The ſecond thing Mr. *Robins* excepts againſt, is my aſſerting, that * "The "Courſe taken by Sir *Iſaac Newton* to find the "*difference* of variable quantities, though not ri- "gorouſly geometrical in the higher caſes, yet "approaches nearer to geometrick rigour than "the method uſed by Monſ. *Leibnitz*."

Here Mr. *Robins* does not diſpute, that Sir *Iſaac Newton's computation may come nearer the value of any quantity ſought after than that of* Leibnitz. But he ſays, that *conſidered as a medium of demonſtration to determine the* ABSOLUTE *value of ſuch quantity, both will be totally, and therefore equally void of geometrick rigour.*

Very true. But I never conſidered this courſe taken by Sir *Iſaac Newton, as a medium of demonſtration to determine the* ABSOLUTE *value*; all that is pretended by it, is to determine the value, not abſolutely, nor in geometrick rigour, but *quam proximè*; to come nearer to the abſolute, geometrick,

* Republick of Letters *for* April, p. 327.

trick, rigorous value, that one can come, by following the method of Monſ. *Leibnitz.*

In conſidering the letters *a*, *b* and *c* in this acceptation, as repreſenting the differences of Monſ. *Leibnitz*, I had an eye not only to the paſſage then quoted, *Utiuſque* (*ſc. methodi*) *fundamentum continetur in hoc lemmate*; but to the following paſſage of the Introduction to the *Quadratures*. *Fluxiones ſunt* QUAM PROXIME *ut fluentium augmenta æqualibus temporis particulis quam minimis genita*; *&*, *ut* ACCURATE *loquar*, *ſunt in prima ratione augmentorum naſcentium.* The *quam proximè* has a view to Mr. *Leibnitz*'s method, and *accuratè* to his own.

XL. The next and laſt thing objected againſt, is my ſaying that † Sir *Iſaac*'s Caſ. 1. of this *lemma* is naturally to be underſtood in one of theſe two ſenſes, that the ſides of the rectangle flow either uniformly, or proportionably.

This, Mr. *Robins* ſays, * "cannot be the natu-
"ral interpretation of that caſe, becauſe it is im-
"mediately quoted to prove the ſecond caſe,
"where the augmentation is confeſſedly diffe-
"rent."

But I take the natural ſenſe of the firſt caſe to be that ſenſe it will appear in to a careful reader, upon peruſing that caſe alone, though he never reads the ſecond caſe at all. Mr. *Robins* ſeems to think Sir *Iſaac Newton* ſo bad a writer, that his firſt caſe is not to be underſtood, till we have read the ſecond; nor his propoſitions to be intelligible, till we read the demonſtration. I think otherwiſe; but I muſt own, *it often requires careful attention.* How to apply the firſt caſe, when

† Republick of Letters *for* November, p. 392.
* Repub. of Let. for *Apr.* p. 327.

underſtood in what I take to be its natural ſenſe, to caſe the ſecond, *where the augmentation is confeſſedly different*, has been clearly ſhown in the *Republick of Letters* for *November* laſt.

Mr. *Robins* goes on thus, " Nor can any rea-
" ſon be aſſigned to ſhew, why it ſhould be thus
" underſtood."

But I apprehend the reader will find clear and ſufficient reaſons for thus underſtanding it, if he is pleaſed to peruſe what I have ſaid upon this *lemma*, in the *Republick of Letters* for *November* laſt, where he will find Sir *Iſaac Newton*'s demonſtration very diſtinctly and particularly explained, not only when moments are taken in the ſenſe I underſtand them in, but alſo in the ſenſe Mr. *Robins* contends for, in taking moments for finite quantities.

To what is there delivered, it may, for Mr. *Robins*'s farther ſatisfaction, be added, that, in this demonſtration, Sir *Iſaac Newton* manifeſtly conſiders three different points of time: One, in which the ſides of the rectangle are of the magnitudes, A $-\frac{1}{2}a$ and B $-\frac{1}{2}b$; A ſecond, in which they are of the magnitudes, A and B; And a third, in which they are of the magnitudes A $+\frac{1}{2}a$ and B $+\frac{1}{2}b$.

Between the firſt and ſecond point of time, he ſuppoſes one half of the moment a, and one half of the moment b to be gained: And between the ſecond and third point of time he likewiſe ſuppoſes $\frac{1}{2}a$ and $\frac{1}{2}b$ to be gained.

Now theſe ſuppoſitions may be taken in two different ſenſes: One, which ariſes from the words of the demonſtration, only, without introducing any foreign conſideration not expreſſed in that demonſtration; and this may be called the natural ſenſe: The other, beſides the words of the demonſtration,

monſtration, requires the introducing the method of firſt and laſt proportions, which though delivered before by Sir *Iſaac Newton*, yet is not mentioned in this demonſtration.

If we take the ſuppoſitions in the firſt ſenſe, each half of the moment *a* will be actually contemporaneous with each half of the moment *b*, *i. e.* the ſides of the rectangle muſt flow either uniformly or proportionably.

If we take them in the ſecond ſenſe, the ſides may flow in any manner whatſoever, and yet, by help of the method of firſt and laſt proportions, we may conceive the halves of the moments to be contemporaneous.

But perhaps Mr. *Robins* would have liked my reaſoning better, if I had ſaid, we muſt take *a* B + *b* A for the moments, *becauſe ſo much only is required for determining the ultimate ratio*; Sir *Iſaac Newton* has taken ſuch a method, *becauſe by this means the ſuperfluous rectangle is ſooner diſengaged from the demonſtration: It is convenient, by neglecting ſome ſuperfluous part of the increments, or by a proper addition to them, to form new quantities; the ſuperfluous terms are rejected out of the increment of the power*, &c.

I muſt needs ſay, this is an eaſier way of reaſoning, than that I have taken: But Αἰδέομαι Τρῶας καὶ Τρωάδας, and moſt of all I ſhould fear the *Author of the Analyſt*. For he, though he would readily grant me, that *it is* CONVENIENT *to neglect ſome ſuperfluous part of the increments, to reject the ſuperfluous terms*; though he might be candid enough to ſay, *The caſe indeed is difficult*; *There can be nothing done till you have got rid of* them; yet, I apprehend, he would not ſtick to tell me, that, although *the final cauſe or motive to this proceeding is very obvious*; yet, *it is not ſo obvious or eaſy to*

explain

explain a juſt and legitimate reaſon for it, or ſhew it to be geometrical.

I muſt confeſs, I ſhould not know what to reply. But it matters not. It is time for us two to give over our weak and fruitleſs contention. Mr. *Robins* now appears in the field:

Mox ambos ſua fata manent majore ſub hoſte.

XLII. Having now replied to all that concerns my ſelf, in this *Diſſertation* of Mr. *Robins*, I ſhall only take notice, that the latter part of it, for about eight pages, is taken up with a vindication, ſhall I ſay, or accuſation? cenſure, or encomium of Sir *Iſaac Newton?* Mr. *Robins* ſhews that reſpect to the memory of this great man, that he will ſuffer no body to attack his reputation, but himſelf. He defends him againſt the *Author of the Analyſt*; but, at the ſame time, he points out other faults in Sir *Iſaac*, which that *Author* never thought of. How happy was it that theſe two Writers did not attack him at the ſame time, were not joined in alliance againſt him. It muſt have been equally dangerous, as if *Philip* had joined his arms to thoſe of *Hannibal*, and not waited the event of the battle at *Zama*.

XLIII. Mr. *Robins* gives us to underſtand, that Sir *Iſaac Newton* was at firſt an errant Indiviſibleiſt: Afterwards indeed he ſaw and renounced his errors. *He had ſufficiently ſeen the abuſes, that had been made of the doctrine of infinitely little quantities. He looked on no demonſtration as valid, that was built on thoſe abſurd principles. He was fully appriſed of the real imperfections of indiviſibles: Yet he ſcarce condemned them*

them himſelf, and frequently made uſe of expreſſions peculiar to them: He ſtill allowed himſelf ſome uſe of infinitely little quantities: He has not *demonſtrated the proportions of fluxions according to the accurate methods of the ancients.*

XLIV. I muſt needs ſay, after this heavy charge, Mr. *Robins* has the goodneſs to make ſome excuſes for Sir *Iſaac Newton.* He was *very young:* He was too *modeſt*, and *as it were fearful:* He had not *read the ancients.* Very kind and tender truly! I ſhould be glad I had as much to ſay for Mr. *Robins:* But for Sir *Iſaac Newton* theſe apologies may as well be let alone.

That he was once *very young*, is not denied. But the youth of that great man was ſuperior to the mature age of the reſt of mankind. He had made the greater part of his amazing diſcoveries by the year 1666, at the end of which year he was four and twenty.

That Sir *Iſaac Newton*'s *modeſty is ſurprizing* to Mr. *Robins*, I do not wonder. But this *ſurprizing modeſty, and as it were fearfulneſs*, did not hinder him from *cenſuring the method of indiviſibles*, though *in the ſofteſt manner.* Notwithſtanding this *modeſty and fearfulneſs, he diſcovered his diſlike to indiviſibles*, he *condemned them*; he *ſhewed a way to avoid their imperfections*; he *has given us a caution in order to prevent miſtakes about many of theſe forms of ſpeech.*

If he did not *read the ancients*, and conſequently could not *from thence be enabled to have demonſtrated the proportions of fluxions according to their accurate methods*; yet Mr. *Robins* tells us, *he did much more, by finding out one of his own, more compendious than theirs, and equally geometrical*; one that is *an abbreviation and improvement of the form*

of

of demonſtrating uſed by the ancients on the like occaſions; one, *which is much more conciſe than the method of demonſtrating uſed in theſe caſes by the ancients, yet is equally diſtinct and concluſive*. So that he ſeems to have ſtood in no great need of the ancients, ſince he ſo much improved and outdid *their accurate methods*.

But why does Mr. *Robins* think, that Sir *Iſaac Newton* had not read the ancients? I find him often mentioning the geometry of the ancients, as, *conſonum eſt geometriæ veterum, more veterum geometrarum, &c.* And the character he gives in the *Scholium* to the firſt ſection of the *Principia*, of theſe ſame *accurate methods*, ſeems to be ſo juſt, that I think he muſt be well acquainted with them. *Præmiſi hæc lemmata, ut* EFFUGEREM TAEDIUM *deducendi* PERPLEXAS *demonſtrationes ad abſurdum more veterum Geometrarum.* To avoid cavil, I muſt take notice, that Sir *Iſaac* was by ſome means or other prevailed upon, to change the word *perplexas* into *longas*, in the laſt edition of the *Principia*. This alteration ſeems to be copied in the following paſſage of Mr. *Robins*. "Sir *Iſaac Newton* by his doctrine of prime "and ultimate ratios, has found out the proper "medium, whereby to avoid the impoſſible no-"tion of indiviſibles on the one hand, and the "*length* of exhauſtions on the other."

Now let all thoſe, who had the honour of Sir *Iſaac Newton*'s acquaintance, and were witneſſes to the modeſty, integrity and veracity of that Great Man, who was ſo ſingularly *Vir antiqui moris*, let them I ſay judge, whether he would have talked in this manner, to make the world believe he had read the works of the ancient Geometricians, if in reality he had not read them. But why do I ſay, let them judge? They all know,

know, from a thouſand inſtances, that he had carefully read them, and was perfectly maſter of them.

XLV. To conclude, if Mr. *Robins will not pretend to defend* Sir *Iſaac Newton* without making ſuch ſort of excuſes and apologies for him; if without *interpreting Sir Iſaac's expreſſions ſuitably to* his own *repreſentation of this doctrine*, he *acknowledges himſelf totally incapable of reconciling the method of prime and ultimate ratios with the character, the author himſelf has given of it*; I think it might be as well to let that defence alone, to ſuffer Sir *Iſaac Newton* to ſtand or fall by the merit of his own works. For my part, I am verily perſuaded, to uſe once more the words of Mr. *Robins*, that *all the abſurdity of expreſſion, and all the inconſiſtency with himſelf charged on Sir* Iſaac Newton *by the Author of the Analyſt*, or any body elſe, *ariſes wholly from miſinterpretation*, or miſunderſtanding him.

E tenebris tantis tam clarum extollere lumen
Qui primus potuiſti, & fluxu expendere curvas;
Te ſequor, O Britonum gentis decus, inq; tuis nunc
Fixa pedum pono preſſis veſtigia ſignis,
Certandi cupidus nequaquam, at propter amorem,
Quod te imitari aveo. Quid enim contendat hirundo
Cycnis? Aut quidnam tremulis facere artubus hoedi
Conſimile in curſu poſſint, ac fortis equi vis?
Tu Pater, & rerum Inventor: Tu patria nobis
Suppeditas præcepta; tuiſque ex, Inclute, chartis,
Floriferis ut apes in ſaltibus omnia libant,
Omnia nos itidem depaſcimur aurea dicta,
Aurea, perpetuâ ſemper digniſſima vitâ.

ERRATA. R. of L. for July, p. 52. lin. 14. r. *genitæ*. R. of L. for Auguſt, p. 121. lin. 20. r. *analogous*. A pag. 164. lin. 17. ad pag. 167. lin. penultim. dele *omnia*.

MY Readers may very juſtly expect an Apology for this Number, wherein they will find no Account of Books, which ought indeed to be the principal Subject, nor any thing but what's peculiar to the Mathematicians. As I aſſure them they ſhall hereafter have no Occaſion to complain upon this Head, ſo I beg leave to lay before them the Reaſon of my preſent Conduct, which I intreat 'em to excuſe without pretending to juſtify. They are all appriſed of a Controverſy concerning Sir *Iſaac Newton*'s Doctrine of Fluxions, which two very learned Antagoniſts have carried on for ſome time paſt in theſe Papers: It is upon a very ſublime and uſeful Topic, and the Diſputants, however they happen to diſagree and miſtake one another, are both of them eminently Maſters of it: and as their Diſcourſes have afforded a noble Entertainment to many Gentlemen of the moſt diſtinguiſhed Skill in the Science; ſo their chuſing to communicate their ſeveral Sentiments to the Public in this Work, has done me a ſingular Honour, and laid me under great and equal Obligations to them both: and nothing but the Hazard of a general Offence, which I hope I ſhall for this time eſcape, could prevail on me to refuſe either of their Requeſts. It happened that ſome Expreſſions of the anonymous Writer in his *Conſiderations, &c.* publiſhed in our laſt Month, were thought injurious by the ingenious Mr. *Robins*, who regarded them as perſonal Reflections, to which he could not refuſe himſelf the Juſtice of an immediate Reply, and as he drew one up with the utmoſt expedition, ſo he demanded a Place for it in my next *Republick*.